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The Timing of Government Debt Reductions in the Presence of Inequality*

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July 11, 2020

Abstract

In order to reduce the government debt, should we increase the consumption tax early or late? This paper uses the incomplete market model to assess the effect of a delay of a consumption tax hike. The result shows the different welfare effect on households with different asset holdings: Poor people prefer early restructuring as late restructuring require a larger increase in consumption tax, and Rich people prefer late restructuring because it increases the interest rate. The overall change in the social welfare is determined by the endogenous distribution of assets.

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1 Introduction

Japan's public debt has been increasing for a long time. According to the IMF Global Debt Database, the public debt to the gross domestic product (GDP) ratio exceeds 200% in 2009 and in 2018 it is around 237%. Figure 1 shows the historical data of the government debt to GDP ratio in Japan. To finance the increase in the social security cost without increasing the government debt further, the Japanese government increased the consumption tax from 5% to 8% in 2014 and to 10% in October 2019. The latter consumption tax hike was originally scheduled in 2015 but postponed twice to 2019.

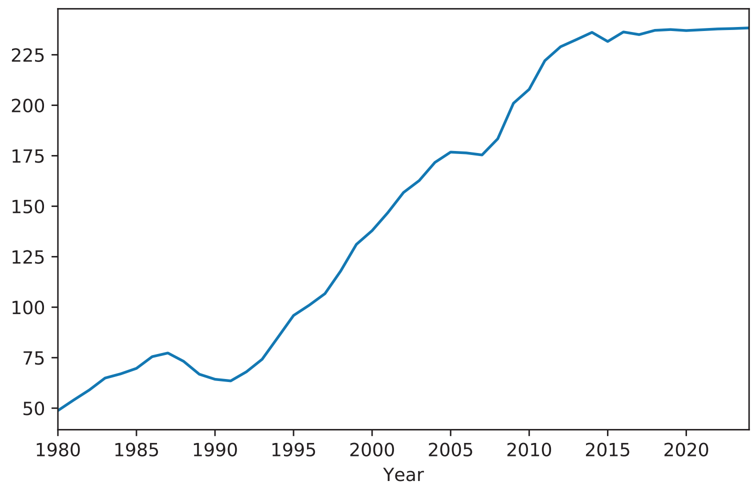

Figure 1: Ratio of gross government debt to gross domestic product. Source: IMF Global Debt Database. The data from 2019 are estimated by the IMF.

Should we increase the consumption tax early or late if we want to decrease the government debt to GDP ratio? Who benefits from early debt restructuring? This paper uses an incomplete market model with government debt to answer these questions. Under the incomplete market, an endogenous distribution of household assets arises, which allow us to explore the effect of the timing of taxation on households with different asset profiles. In addition, we analyze the transition path associated with the consumption tax hike to take into account the short-run effect as well as the long-run effect.

After calibrating the model to the Japanese data, we computed the transition dynamics associated with a sudden increase in the government purchase and early or late increases in the consumption tax. The result shows the different welfare effect on households with different asset holdings: Poor people prefer early restructuring as late restructuring requires a larger increase in consumption tax. However. rich people prefer late restructuring because it increases the interest rate. The overall change in the social welfare is determined by the endogenous distribution of assets. Under our calibration, 71.07% of people prefer early restructuring.

The rest of the paper is organized as follows. Section 2 reviews the related literature. Section 3 presents the model. Section 4 describes the calibration and numerical results. Section 5 concludes.

2 Literature

This paper is related to the literature on government debt in incomplete market models. Aiyagari and McGrattan (1998) studied the optimal quantity of debt in the incomplete market model in which government debt may provide additional insurance as a way to safeguard from the future risk. Floden (2001) evaluated the role of government debt when the transfer also changes and found that the welfare effect of government debt is small. Peterman and Sager (2018) used the overlapping generation heterogeneous agent model to evaluate the optimal quantity of government debt. They found that the optimal quantity of government debt is negative, that is, the government should be a net saver. In our paper, although we implement a simulation which tries to decrease the government debt, our focus is on the timing of consumption tax, rather than the amount of government debt.

Although these papers focus on the stationary equilibrium, there are papers to study the transitional dynamics in the heterogeneous agent model. Conesa and Krueger (1999) and De Nardi, Imrohoroğlu, and Sargent (1999) are the early examples. In terms of government debt, Desbonnet and Weitzenblum (2012) consider a government which tries to maximize the social welfare by changing the amount of government debt, and found that the government tend to be trapped in the large amount of debt because the short run transitional effect of the reducing the government debt is costly. Röhrs and Winter (2017) also considers a reduction in the government debt in the heterogeneous agent model, and found that agents prefer that the tax burden is postponed into the future.

This paper is also related to the study of the Japanese fiscal consolidations. Braun and Joines (2015) studied the implications of the aging society on the fiscal situation of the Japanese government using an overlapping generation model. Hansen and İmrohoroğlu (2016) builds a neoclassical growth model with government debt in the utility function to study alternative ways to finance projected increase in the government expenditure and stabilize the debt to GDP level. Kitao (2018) considers the case where fiscal consolidation occurs in the future but the timing is uncertain. Although we study the fiscal situation of Japan, our paper focus on the effect on households with different asset profiles, not only the aggregate economy. One exception is Nakajima and Takahashi (2017). They calibrate the Aiyagari (1994)-style model to the Japanese economy and compute the debt to GDP ratio which maximize utilitarian social welfare. Our paper studies the transitional dynamics associated with consumption tax hikes. Nakajima and Takahashi (2019) uses the U.S. data and concludes that the consumption tax has a weak insurance effect.

3 The model

Our model is based on Aiyagari (1994) where households face an idiosyncratic shock to their labor productivity, and there is no insurance market for the shock. To insure against this shock, households can save their resource for the future by lending to other agents, including firms and government. As the labor productivity shock is idiosyncratic, some lucky households keep getting good shock and accumulate assets, while unlucky households keep getting bad shock and reach the borrowing constraint eventually. This will lead to the non-trivial endogenous distribution of household assets. This feature allows us to study the implication of a change in the consumption tax on different households in an dynamic general equilibrium framework.

3.1 Environment

Time is discrete and continue forever $t = 0, 1, 2, \ldots$. There are three types of sectors in this model, households, firms, and government. There are three goods, labor, asset, and final good which can be used for consumption and investments. For each period we treat the final good as numeraire.

Now we describe how these sectors behave in the following section.

3.1.1 Households

There is a continuum of households with measure one in this economy. Each period households receive one unit of time and divide it to leisure l_t and labor $1-l_t$. Their labor productivity ϵ_t is subject to an idiosyncratic shock, and there is no insurance market for the shock. The effective labor supply of a household with the labor productivity ϵ_t is then $\epsilon_t(1-l_t)$. Households can save in the form of state non-contingent claim which earns the interest rate r_t. They can also borrow at the rate r_t, although their borrowing is subject to a borrowing constraint which requires that the borrowing of a household should be smaller than or equal to $\underline{a} \geq 0$.

The government uses consumption tax $\tau_{c,t}$, labor income tax $\tau_{w,t}$, and capital income tax $\tau_{r,t}$ to collect tax revenues. In addition, the government pay transfers Tr_t to all households.

The budget constraint of a household in period t is given by

$$(1+\tau_{c,t})c_t + a_{t+1} = [1 + (1-\tau_{r,t})r_t]a_t + (1-\tau_{w,t})W_t\epsilon_t(1-l_t) + Tr_t \qquad (1)$$

where c_t denotes consumption, a_t is the asset holdings, and W_t is the wage.

Households chooses a sequence of consumption, asset holdings, and leisure $(c_t, a_{t+1}, l_t)_{t=0}^{\infty}$ to maximize their lifetime expected utility

$$E_0\left[\sum_{t=0}^{\infty}\beta^t u(c_t, l_t)\right] \tag{2}$$

subject to the budget constraint (1) and the borrowing constraint

$$a_{t+1} \geq \underline{a}. \tag{3}$$

For later use, we define $\mu_t(a, \epsilon)$ as a probability density function of households in the state (a, ϵ).

3.1.2 Firms

Firms use capital K_t and labor N_t to produce the final goods Y_t through the Cobb-Douglas production technology

$$Y_t = K_t^{\theta}(z_t N_t)^{1-\theta} \tag{4}$$

where z_t is the labor-augmenting technological progress. z_t grows at the constant rate g exogenously, $z_{t+1} = z_t(1+g)$. Firms pay the wage W to hire labor and the real interest rate plus depreciation $r + \delta$ to borrow capitals.

3.1.3 Government

The government collects taxes (τ_c, τ_w, τ_r) and issues government debts B_{t+1} to finance the repayment of the debt plus interest payment, $(1 + r_t)B_t$, and the government purchases G_t. The government budget constraint is then given by

$$(1 + r_t)B_t + G_t = B_{t+1} + \tau_{c,t}C_t + \tau_{r,t}r_t K_t + \tau_{w,t}W_t L_t. \tag{5}$$

3.2 Equilibrium

Now, we describe the optimization problem each agent faces and present the definition of the equilibrium.

3.2.1 Household problem

Households choose consumption, leisure hours, and asset holdings to maximize their expected utility:

$$\max_{\{C_t, A_{t+1}, l_t\}} E\left[\sum_{t=0}^{\infty}\beta^t \frac{(C_t^{\eta} l_t^{1-\eta})^{1-\mu}}{1-\mu}\right] \tag{6}$$

Because of the exogenous productivity growth, we need to de-trend this problem to transform it into a stationary dynamic programming problem. The de-trended problem (see Appendix for details) is

$$V(a,\epsilon) = \max_{a',c,l}\{u(c,l) + \tilde{\beta}\sum_{\epsilon'} P(\epsilon'|\epsilon)V(a',\epsilon')]\} \tag{7}$$

$$\text{s.t.} \quad (1+\tau_c)c + (1+g)a' = (1-\tau_w)w\epsilon(1-l) \tag{8}$$

$$+ (1 + (1-\tau_r)r)a + \chi,$$

$$a' \geq \underline{a}, \quad l \in [0,1], \;\; c \geq 0. \tag{9}$$

where $\chi \equiv TR/Y$ is the government transfer and

$$\log(\epsilon') = \rho\log(\epsilon) + e, \quad e \sim N(0,\sigma^2). \tag{10}$$

3.2.2 Firms

Firms choose capital and labor to maximize their profit:

$$\max_{K,L} K^{\alpha}(zL)^{1-\alpha} - WL - (r+\delta)K \tag{11}$$

The first order condition is

$$W = z(1-\alpha)\left(\frac{K}{zL}\right)^{\alpha-1} \tag{12}$$

$$r = \alpha\left(\frac{K}{zL}\right)^{\alpha} - \delta. \tag{13}$$

3.2.3 Government

The government with initial debt B_t needs to finance both the interest payment of the debt $(1+r_t)B_t$ and government expenditure G_t and government transfers TR_t. To do so, it uses consumption tax $\tau_{c,t}$, income tax $\tau_{y,t}$, asset tax $\tau_{a,t}$, and issuance of new debts B_{t+1}.

The government faces the budget constraint of the form

$$G_t + TR_t + (1+r_t)B_t = \tau_{y,t}(r_tA_t^+ + W_tL_t) + \tau_{c,t}C_t + \tau_{k,t}A_t^+ + B_{t+1} \tag{14}$$

where $A_t^+ \equiv \int_0^\infty a d\mu_t(a,\epsilon)$ is the aggregate asset excluding borrowing from the household sector and $C \equiv \int c_t(a,\epsilon)d\mu_t(a,\epsilon)$ is the aggregate consumption.

3.2.4 Markets

The asset and labor market clearing conditions are given by

$$L = \int \epsilon[1 - l(a,\epsilon)] \; d\mu(a,\epsilon) \tag{15}$$

$$K + B = \int A \; d\mu(A,\epsilon) \tag{16}$$

In equilibrium, the goods market clearing condition is satisfied automatically as long as other equilibrium conditions are satisfied by the Walrus' law.

3.2.5 Distributions

Given the distribution of households in period 0, μ_0, the distribution evolves over time as follows:

$$\mu_{t+1}(a', \epsilon') = \sum_a \sum_\epsilon \mathbf{1}\{a'_t(a, \epsilon) = a'\} P(\epsilon'|\epsilon)\mu_t(a, \epsilon) \tag{17}$$

where $\mathbf{1}\{\dot{}\}$ is the indicator function. This equation describes the measure of households in state (a', ϵ') at period t. Consider a household whose state is (a, ϵ). That household moves to state (a', ϵ') with probability $P(\epsilon'|\epsilon)$ if its policy function is consistent with a' $(a'_t(a, \epsilon) = a')$. The measure of households in state (a, ϵ) is $\mu_t(a, \epsilon)$. Then we can take the summation over (a, ϵ) to obtain the total measure of households in (a', ϵ').

3.2.6 Detrended stationary equilibrium

Since this model has exogenous technology growth, the economy will grow at a constant rate in the stationary equilibrium. Details about how to detrend the model are presented in the appendix.

Given a fiscal policy $\boldsymbol{\tau} = (\tau_w, \tau_G, \tilde{B})$, a stationary equilibrium is $(V, \tilde{a}', \tilde{W}, r, \mu, \tilde{K}, L, \tau_c)$ such that the following is true.

Consumer optimization: V solves the Bellman equation and a' is the associated policy function:

$$V(\tilde{a}, \epsilon, \boldsymbol{\tau}) = \max_{\tilde{c}, \tilde{a}', l} \left\{ \frac{(\tilde{c}^\eta l^{1-\eta})^{1-\mu}}{1-\mu} + \tilde{\beta} E[V(\tilde{a}', \epsilon', \boldsymbol{\tau})] \right\} \tag{18}$$

$$\text{s.t.} \quad \tilde{c} + (1+g)\tilde{a}' = (1+r)\tilde{a} + \tilde{W}(1-l)\epsilon + \chi. \tag{19}$$

Producer optimization: Prices (r, w) and aggregate variables (k, L) are consistent with the firm's optimization:

$$W = z(1-\alpha)\left(\frac{K}{zL}\right)^{\alpha-1} \tag{20}$$

$$\tilde{W} \equiv \frac{W}{z} = (1-\alpha)\left(\frac{\tilde{K}}{L}\right)^{\alpha-1} \tag{21}$$

$$r = \alpha\left(\frac{\tilde{K}}{L}\right)^{\alpha} - \delta. \tag{22}$$

Market clearing : The capital and labor market clear:

$$L = \int \epsilon[1 - l(a, \epsilon)]\, d\mu(a, \epsilon), \tag{23}$$

$$\tilde{A} = \tilde{K} + \tilde{B}. \tag{24}$$

Stationarity: The distribution of agents μ is constant over time:

$$\mu(a', \epsilon', \boldsymbol{\tau}) = \sum_a \sum_\epsilon \mathbf{1}\{a(a,\epsilon) = a'\} P(\epsilon'|\epsilon)\mu(a, \epsilon, \boldsymbol{\tau}). \tag{25}$$

Government budget constraint: The government budget constraint is satisfied:

$$\tau_y(r\tilde{K} + \tilde{W}L) + \tau_c\tilde{C} = \gamma + \chi + (r - g)\tilde{B}. \tag{26}$$

3.3 Transition dynamics

The stationary equilibrium is a convenient way to summarize the long run response of the economy to the policy change. However, when we consider fiscal consolidation, we should be careful about using the stationary equilibrium. If we compare the stationary equilibrium with $b = 0.6$ and $b = 1$, we implicitly assume that we can move between these equilibria without incurring the short run cost of the reduction in government debt; that is, to decrease the debt to GDP ratio by 40%, the cost associated with it will be enormous. Here we describe the transition dynamics associated with the change in the exogenous parameter (government expenditures to GDP ratio, γ) and associated policy changes.

Let T denote the time when the economy will be back to the new stationary equilibrium. After that period, the economy is in the new stationary equilibrium forever, so we can summarize the transition dynamics as a sequence with length T. Suppose that the economy is in the stationary equilibrium associated with $\boldsymbol{\tau}^{ini}$ initially. We are interested in the transition dynamics with the terminal condition $\boldsymbol{\tau}^{terminal}$.

Given a fiscal policy $\boldsymbol{\tau}_t = (\tau_{y,t}, \gamma_t, \chi_t, \tilde{B}_t)$, a transition equilibrium is $(V_t, \tilde{a}'_t, \tilde{W}_t, r_t, \mu_t, \tilde{K}_t, L_t, \tau_{c,t})_{t=0}^T$ such that the following is true.

Consumer optimization: V solves the Bellman equation and a' is the associated policy function:

$$V_t(a, \epsilon) = \max_{a',c,l}\{u(c, l) + \tilde{\beta} \sum_{\epsilon'} P(\epsilon'|\epsilon)V_{t+1}(a', \epsilon')]\} \tag{27}$$

$$\text{s.t.} \quad (1 + \tau_{c,t})c + (1 + g)a' = (1 - \tau_{y,t})w_t\epsilon(1 - l) \tag{28}$$

$$+ (1 + (1 - \tau_{y,t})r)a + \chi,$$

$$a' \geq \underline{a}, \quad l \in [0, 1], \; c \geq 0, \tag{29}$$

where $V_T(a, \epsilon) = V(a, \epsilon, \boldsymbol{\tau}^{terminal})$.

Producer optimization: Prices (r, w) and aggregate variables (k, L) are consistent with the firm's optimization:

$$\tilde{W}_t \equiv \frac{W_t}{z_t} = (1-\alpha)\left(\frac{\tilde{K}_t}{L_t}\right)^{\alpha-1} \tag{30}$$

$$r_t = \alpha\left(\frac{\tilde{K}_t}{L_t}\right)^{\alpha} - \delta. \tag{31}$$

Market clearing : The capital and labor market clear:

$$L_t = \int \epsilon[1 - l_t(a,\epsilon)]\ d\mu_t(a,\epsilon), \tag{32}$$

$$\tilde{A}_t = \tilde{K}_t + \tilde{B}_t. \tag{33}$$

Distribution: The distribution of agents μ_t evolves according to

$$\mu_{t+1}(a',\epsilon') = \sum_a \sum_\epsilon \mathbf{1}\{a'_t(a,\epsilon) = a'\}P(\epsilon'|\epsilon)\mu_t(a,\epsilon), \tag{34}$$

with $\mu_0((a,\epsilon) = \mu(a,\epsilon,\boldsymbol{\tau}^{ini})$.

Government budget constraint: The government budget constraint is satisfied:

$$\tilde{G}_t + (1+r_t)\tilde{B}_t = \tilde{B}_{t+1}(1+g) + \tau_{c,t}\tilde{C}_t + \tau_{y,t}(r_t\tilde{K}_t + \tilde{W}_t L). \tag{35}$$

Details about how to compute the transitional dynamics are presented in the appendix.

4 Numerical results

We calibrate the model to Japanese data. Specifically, we used the following parameter values.

In this paper, a period in the model corresponds to a year in the data. In order to capture the recent low interest rate and growth rate in Japan, we set $\beta = 0.991$ and $g = 0.009$ following Nakajima and Takahashi (2017). We use the standard constant relative risk aversion utility function $u(c,l) = [\frac{(c^\eta l^{1-\eta})^{1-\mu}}{1-\mu}]$, with $\mu = 1.5$, $\eta = 0.328$. We assume that agents cannot borrow, that is, $\underline{a} = 0$. To capture the capital labor ratio of 0.3, we set $\alpha = 0.3$. We set $\delta = 0.075$ so the capital output ratio in the stationary equilibrium is around 4.

The income process is assumed to be an AR(1) process of the form

$$\log(\epsilon_{t+1}) = \rho\log(\epsilon_t) + e_t, \quad e_t \sim N(0,\sigma_e^2). \tag{36}$$

Nakajima and Takahashi (2017) used an income process with persistent component. Since we are computing the transition dynamics, we simplify this process by assuming that there is only a transitory component. We set $\rho = 0.9$

which is the number used in Nakajima and Takahashi (2017), and $\sigma_e = 0.226$. This process is discritized into a seven state Markov chain using the method of Tauchen (1986). Finally, for the government policy, we set $\chi = TR/Y = 0.141$, $\gamma = G/Y = 0.13$ for the initial stationary equilibrium and 0.24 for the terminal stationary equilibrium. This number is chosen so that the consumption tax in the initial stationary equilibrium is 8% and the government purchase to GDP ratio is consistent with the projection in Fukawa and Sato (2009) in the terminal stationary equilibrium. For the tax rates other than the consumption tax which is determined endogenously, we set the capital and labor income tax the same to ease the computational burden. We set $\tau_w = \tau_r = 0.34$ following Hansen and İmrohoroğlu (2016).

4.1 Numerical analysis: Stationary equilibrium

We first compute the stationary equilibria for different debt-to-GDP ratios, $b = 0.6$ and $b = 1$. Table 1 reports the aggregate variables in the stationary equilibrium with $b = 0.6$ and $b = 1$. An increase in the debt to GDP ratio leads to higher consumption tax to finance the interest payment. In addition, higher b crowds out the capital: if b increases, to satisfy the asset market clearing condition, either the capital decreases or the interest rate increases so that households want to hold more assets, which decreases the demand for capital.

Figure 2 and 3 shows the stationary distribution of households with specific lbor productivity with $b = 0.6$ and $b = 1$. The numbers in these figures can be interpreted as a number of households with specific amount of assets. Different color of graphs represents different labor productivity. As the aggregate debt increases, the stationary distribution moves to the right, implying that households accumulate more assets.

	N	r	b	τ_c
$b = 0.6$	0.3077	1.46	0.6	8.00
$b = 1.0$	0.3064	1.53	1.0	30.58

Table 1: Comparison of two stationary equilibria

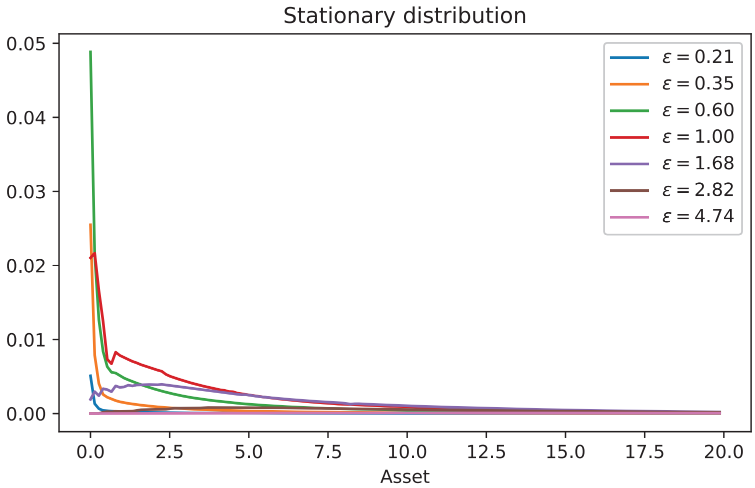

Figure 2: Stationary distribution under $b = 0.6$

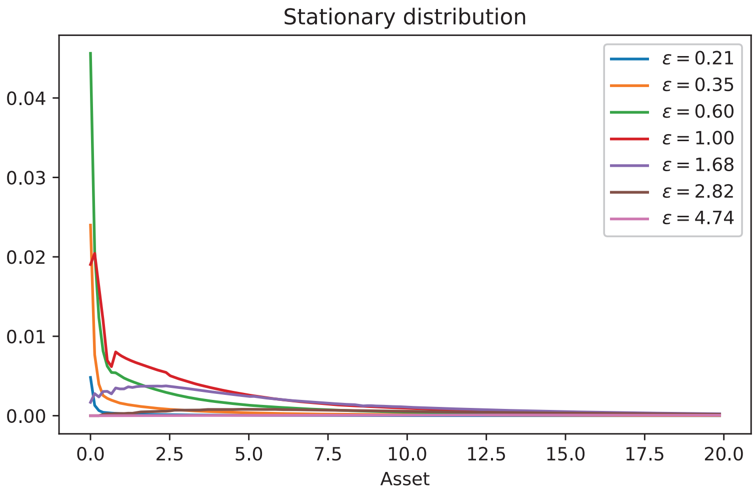

Figure 3: Stationary distribution under $b = 1.0$

4.2 Numerical analysis: Transition dynamics

Now, we compute the transition path in response to a change in the government expenditure and associated tax increases. Here we implement the following simulations.

Suppose that, initially, the economy is at the stationary equilibrium with

$b = 0.6$. At data 0, there is an unexpected increase in government expenditure g. This can be interpreted as the increase in the social security cost. To satisfy the government budget constraint, the government can change either τ_c or increase b_{t+1}. Now, we consider two policy options.

1. Keep $\tau_c = 0.08$ until $b = 1.5$ and then increase τ_c so that $b = 1$ after 40 years.

2. Keep $\tau_c = 0.08$ until $b = 3.0$ and then increase τ_c so that $b = 1$ after 40 years.

The former option corresponds to early fiscal consolidations and the latter late consolidations. In each scenario, the consumption tax $\tau_{c,t}$ after fiscal consolidation is determined to that b decrease at a constant rate and converges to $b = 1$ after 40 years. We now compute the transitional dynamics associated with these two policy options, and ask households in period 0 which policy they prefer.

Figure 4 shows the transition dynamics under this simulation. When the government runs fiscal deficits financed by issuance of government debt, the interest rate rises and the capital decreases as a result of the increase in the demand for assets. After the government started to increase the consumption tax, the government debt decreases and the capital increases. Since the terminal stationary debt to GDP ratio is higher than the initial debt to GDP ratio, the interest rate falls in the long run.

When we compare the early and late fiscal consolidations, we can see that late consolidations require larger consumption tax hikes, as the higher debt-to-GDP ratio and interest rate leads to larger fiscal deficits without consumption tax hikes. However, since it occurs at a later time period, it is not obvious which policy households choose.

Figure 5 reports the difference in welfare between the early and late consolidations. This figure plots

$$V_0(a, \epsilon; \text{early tax increase}) - V_0(a, \epsilon; \text{late tax increase}) \tag{37}$$

for each labor productivity ϵ. If the number is positive, then the household with the state (a, ϵ) obtains higher utility under the early tax increase. This figure shows that poor (in terms of assets) and low productivity households obtain higher utility from the early consolidation while rich and high productivity households obtain higher utility from late consolidation. This is because rich people can receive a higher interest income due to the increase in the government debt while poor households do not receive interest income, and a higher consumption hike hurts them.

Figure 6 plots the voting decision of households with specific labor productivity. This decision is equal to 1 if the value of (37) is greater than or equal to 0, while it is equal to 0 if the value of (37) is less than 0. This function can be interpreted as a voting decision, which is equal to 1 if a household agrees on early tax increase while it is equal to 0 if that household agrees on late tax increase. Figure 6 suggests that poor households vote for early consolidations, while rich households vote for late consolidations. The voting share of early consolidations – which we can obtain by multiplying the results in Figure 2 with those in Figure 6 and summing up – is 71.07%. Here many households vote for early consolidation, because most people are at the lower end of wealth distribution.

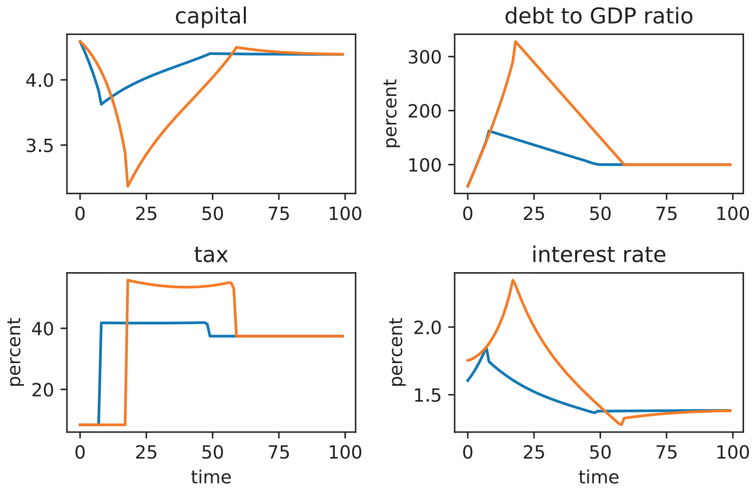

Figure 4: Transition dynamics when tax change is delayed until $b = 1.5$ (blue) and $b = 3$ (orange)

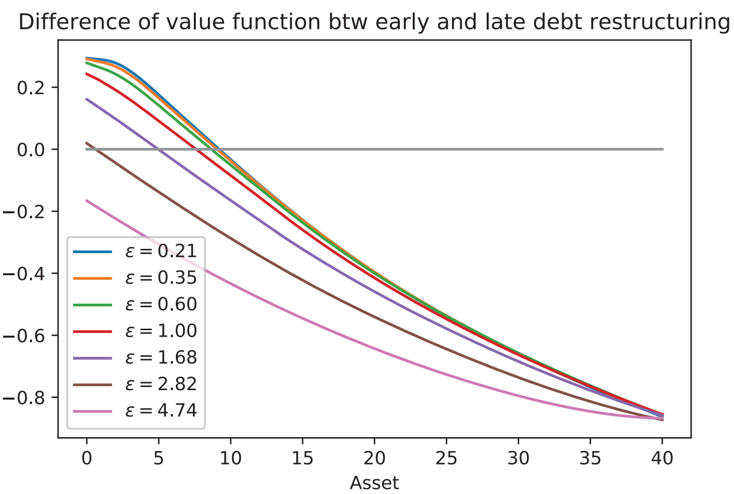

Figure 5: Difference of welfare between early and late consolidations

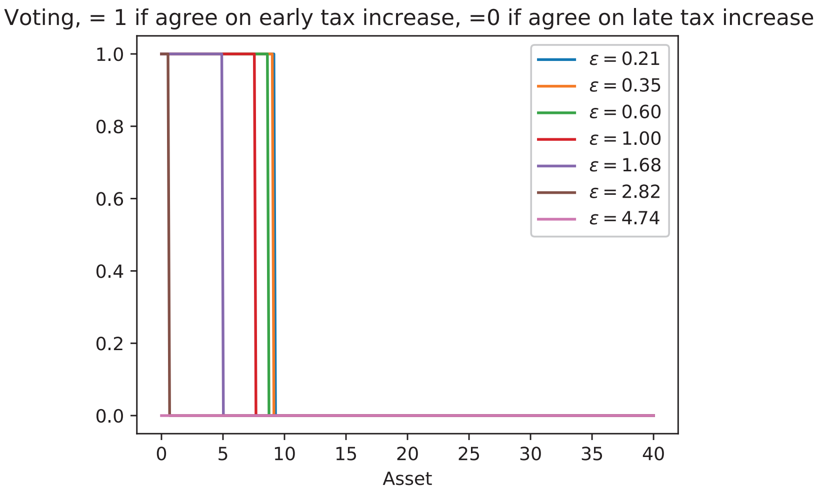

Figure 6: Voting decision for early and late consolidations

4.3 Robustness check 1: Longer tax increase period

The previous simulation assumes that the fiscal consolidation ends within the same time periods for both scenarios. This assumption be strong because with higher debt ($b = 3$), it may take longer time to go back to the new stationary equilibrium. To consider this possibility, suppose that it takes 60 years to finish the debt restructuring with $b = 3$. How do household respond to this restructuring?

Figure 7 shows the transition dynamics under this simulation. In the case with higher debt ($b = 3$), it now takes 60 years to go to the terminal stationary equilibrium. As a result, the tax rate with higher debt in this case is lower than that with short tax increase period in Figure 4. With longer tax increase periods, now the government can smooth the consumption tax.

Figure 8 plots the difference of welfare between early (with 40 periods of tax increase) and late (with 60 periods of tax increase) consolidations. Compared with the case where both tax increase occurs within 40 periods (Figure 5), more people prefer late consolidations. Because of longer tax increase periods and smoothed consumption tax, households can smooth their consumption better than the case with short tax increase periods. When the period of higher tax increases to 60 periods in case 2, the voting share to the option 1 decreases to 54.45%. This is consistent with the finding in Röhrs and Winter (2017) that agents prefer a longer period to reduce government debt.

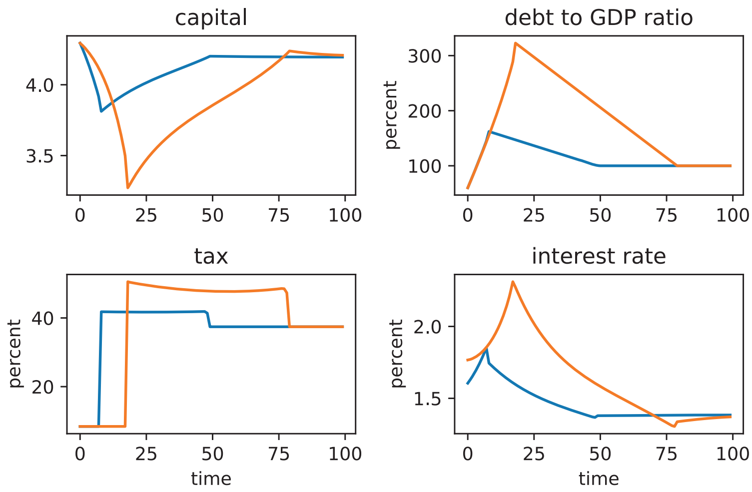

Figure 7: Transition dynamics when the tax change is delayed until $b = 1.5$ with a 40-years recovery period(blue) $b = 3$ with a 60-years recovery period(orange)

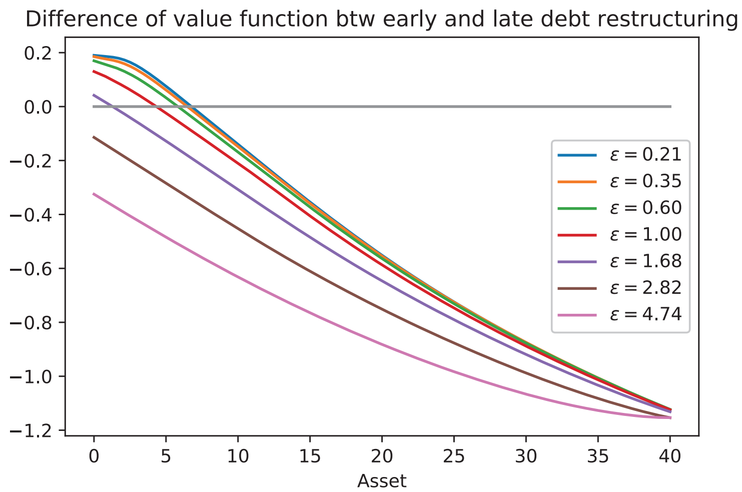

Figure 8: Difference in welfare between the early ($b = 1.5$, 40years) and late ($b = 3$, 60 years) consolidations

4.4 Robustness check 2: Capital tax

The previous simulation assumes that the government can only use the consumption tax to increase its revenue. What happens if the government can also use other taxes? Kobayashi and Ueda (2018) used the capital tax during the debt crisis to reduce the debt burdens. Here we consider a situation where the government uses the capital tax to increase its revenue. In order to keep the long run stationary equilibrium the same, we still use the consumption tax, but since the increase in this tax is limited, the remaining revenue should be financed by capital tax.

Figure 9 plots the transition path with capital tax. In this case, the tax base is broader than the consumption, so the tax rate on capital is small. As a result, the amount of capital is not affected by capital tax so much.

Figure 10 reports the difference of welfare between early and late consolidations when capital tax is used. When the capital tax is available, the result changes. Under capital taxation, although it is still true that the benefit of early consolidation is higher for poor people, now all people prefer late consolidation.

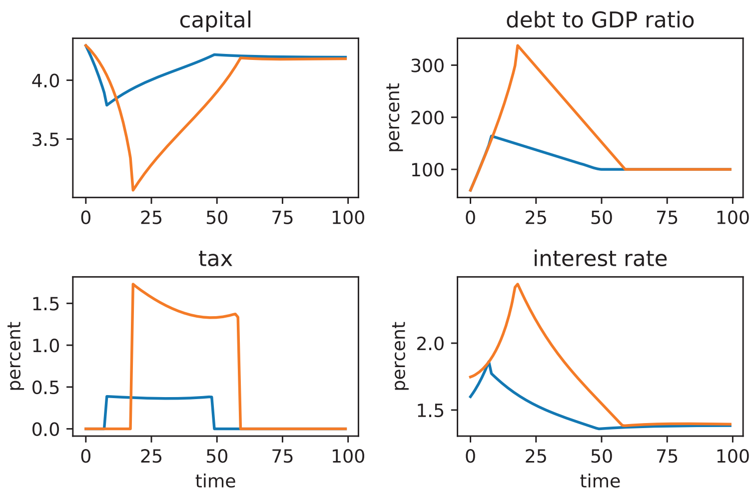

Figure 9: Transition dynamics when the capital tax is available.

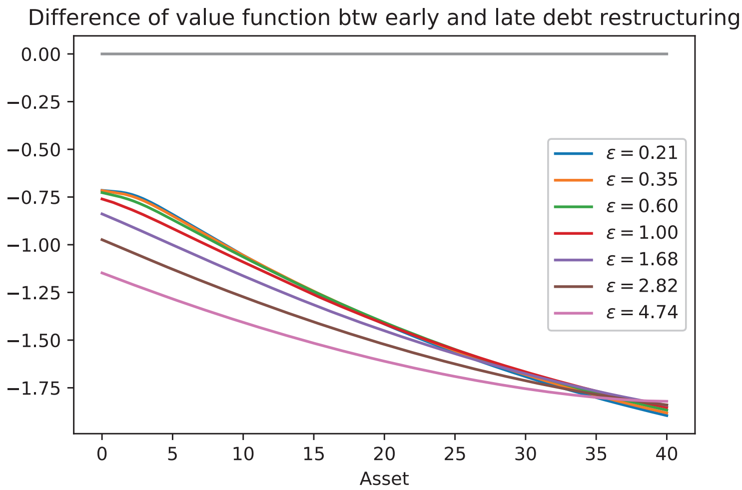

Figure 10: Difference in welfare between early and late consolidations when capital tax is available

Figure 11 plots the transition path with capital tax with short tax increase period (10 periods). In this case, there is a steep increase in capital tax especially with late consolidations. As a result, the amount of capital decrease more than the previous simulations.

Figure 12 reports the difference of welfare between early and late consolidations when capital tax is used with short tax increase periods. When the tax change finishes in 10 periods, then rich people prefer early restructuring. This is because when the tax change finished in 10 periods, the tax rate should increase a lot so that after tax interest rate drops. This didn't happen in the case of 40 periods tax change.

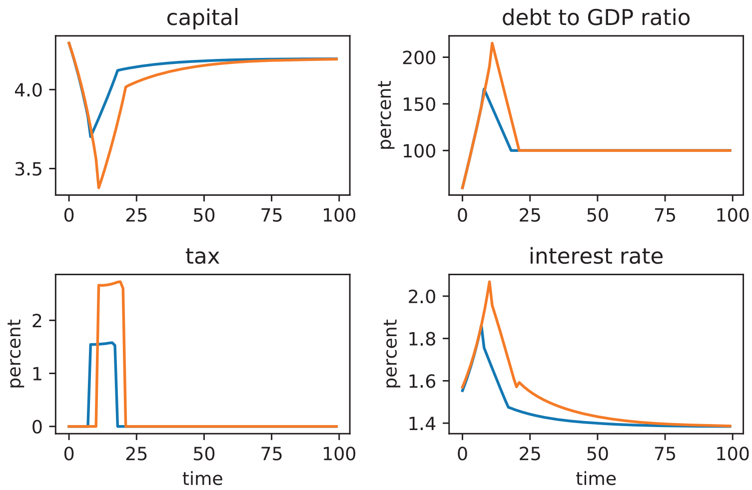

Figure 11: Transition dynamics when the capital tax is available.

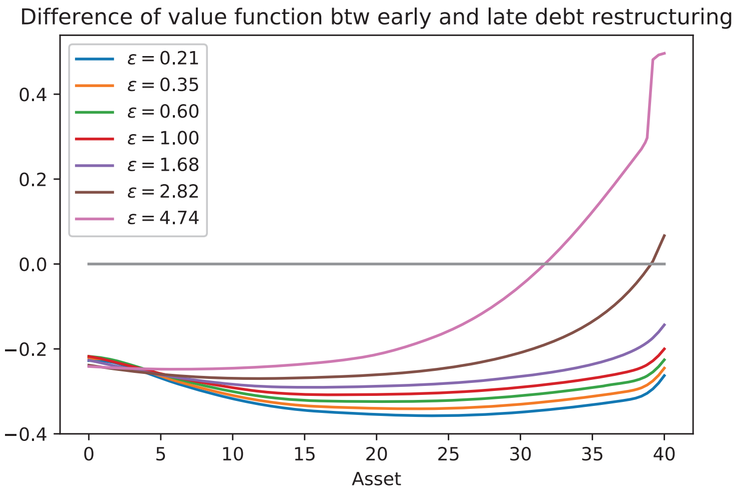

Figure 12: Difference in welfare between early and late consolidations when capital tax is available

4.5 Robustness check 3: Transfer

The analysis so far assumed that the government increases its expenditure at time 0. In this section we analyze the case where the government increase the transfer instead. This can be seen as an increase in the social security, though in reality it is only paid to older generations, not young generations.

Figure 13 plots the transition path when transfer is increased instead of government expenditure. In this case, since inequality decreases due to an increase in government transfers, consumers do not need to save, so the aggregate capital declines. This reduces over-accumulation of capitals and increases utilities of consumers.

Figure 14 the difference of welfare between early and late consolidations when government transfer is changed instead of government expenditure. Even though we change government transfer instead of government expenditure, the result looks qualitatively similar to the case with government expenditure. This is because what matters to this welfare change is the difference of tax rate, and since transfer is increased in both options, it does not change the result significantly. Now the voting share of early consolidations is 72.40%, which is similar to the case with an increase in the government expenditure.

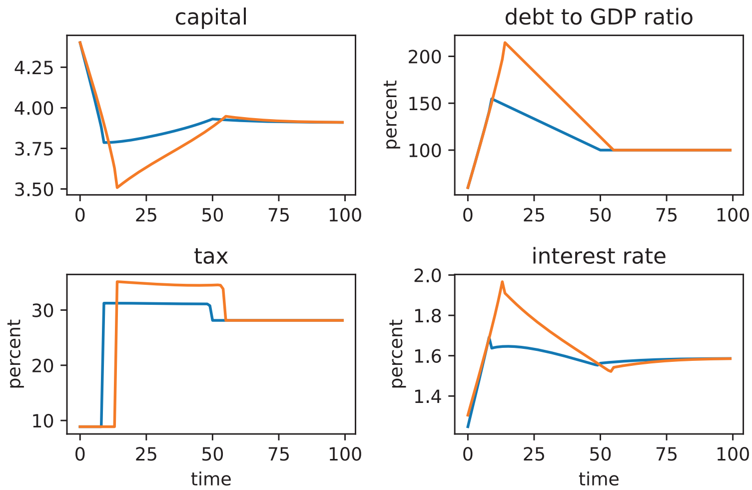

Figure 13: Transition dynamics associated with a change in transfers when tax change is delayed until $b = 1.5$ (blue) and $b = 3$ (orange)

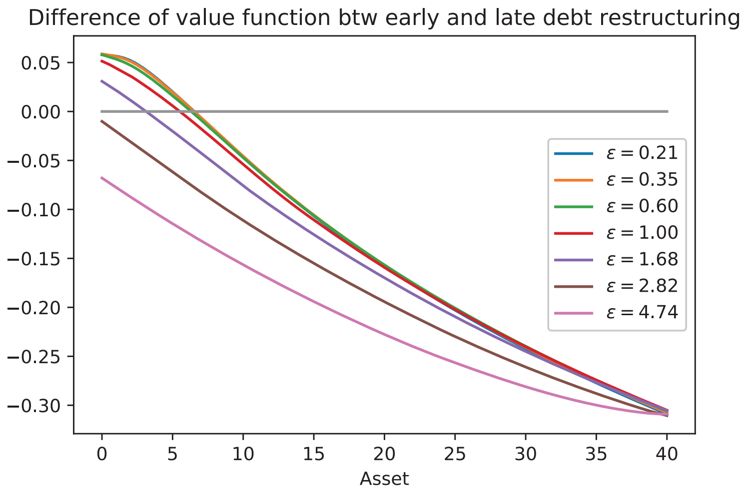

Figure 14: Difference of welfare between early and late consolidations with a change in transfers

5 Conclusion

This paper used an incomplete market model with government debt calibrated to the Japanese data, to see the effect of early and late debt restructuring on households with different asset holdings.

The results show that poor people prefer early restructuring as late restructuring require a larger increase in consumption tax. However, rich people prefer late restructuring because it increases the interest rate. The overall change in social welfare is determined by the endogenous distribution of assets. Under our calibration, 71.07% of people prefer early restructuring.

In this paper we focus on agents who live forever. One of the reason that the government faces the tight budget is an increase in the social security cost, which do not arise in this environment. Introducing overlapping generation structure on top of the incomplete market assumption as in Huggett (1996) and analyse the effect of delaying the fiscal consolidation on people with different asset profiles and different ages is an interesting and policy relevant direction for future research.

References

Aiyagari, S. R. (1994): "Uninsured idiosyncratic risk and aggregate saving," *The Quarterly Journal of Economics*, 109, 659–684.

Aiyagari, S. R. and E. R. McGrattan (1998): "The optimum quantity of debt," *Journal of Monetary Economics*, 42, 447–469.

Badshah, M., P. Beaumont, and A. Srivastava (2013): "Computing equilibrium wealth distributions in models with heterogeneous-agents, incomplete markets and idiosyncratic risk," *Computational Economics*, 41, 171–193.

Braun, R. A. and D. H. Joines (2015): "The implications of a graying Japan for government policy," *Journal of Economic Dynamics and Control*, 57, 1–23.

Conesa, J. C. and D. Krueger (1999): "Social security reform with heterogeneous agents," *Review of Economic dynamics*, 2, 757–795.

De Nardi, M., S. Imrohoroğlu, and T. J. Sargent (1999): "Projected US demographics and social security," *Review of Economic dynamics*, 2, 575–615.

Desbonnet, A. and T. Weitzenblum (2012): "Why Do Governments End Up with Debt? Short-Run Effects Matter," *Economic inquiry*, 50, 905–919.

Floden, M. (2001): "The effectiveness of government debt and transfers as insurance," *Journal of Monetary Economics*, 48, 81–108.

Fukawa, T. and I. Sato (2009): "Projection of pension, health and long-term care expenditures in Japan through macro simulation," *The Japanese Journal of Social Security Policy*, 8, 33–42.

Hansen, G. D. and S. İmrohoroğlu (2016): "Fiscal reform and government debt in Japan: A neoclassical perspective," *Review of Economic Dynamics*, 21, 201–224.

Huggett, M. (1996): "Wealth distribution in life-cycle economies," *Journal of Monetary Economics*, 38, 469–494.

Kitao, S. (2018): "Policy uncertainty and cost of delaying reform: The case of aging Japan," *Review of Economic Dynamics*, 27, 81–100.

Kobayashi, K. and K. Ueda (2018): "Secular stagnation under the fear of a government debt crisis," *Unpublished manuscript.*

Nakajima, T. and S. Takahashi (2017): "The optimum quantity of debt for Japan," *Journal of the Japanese and International Economies*, 46, 17–26.

——— (2019): "The Effectiveness of Consumption Taxes and Transfers as Insurance against Idiosyncratic Risk," *Journal of Money, Credit and Banking.*

Peterman, W. and E. Sager (2018): "Optimal Public Debt with Life Cycle Motives," *mimeo*.

Röhrs, S. and C. Winter (2017): "Reducing government debt in the presence of inequality," *Journal of Economic Dynamics and Control*, 82, 1–20.

Tauchen, G. (1986): "Finite state markov-chain approximations to univariate and vector autoregressions," *Economics letters*, 20, 177–181.

A Appendix

A.1 Detrending

Since the model in this paper contains exogenous growth of technology, we need to detrend endogenous variables to make the environment stationary. Here we detrend the endogenous variable by dividing by z_t which grows at the constant rate, rather than Y_t as in Aiyagari and McGrattan (1998) which grows endogenously.

Let $\tilde{X}_t \equiv X_t/z_t$ denote an endogenous variable divided by the productivity. Now the households' problem can be written as

$$\begin{aligned}
\frac{(c_t^{\eta} l^{1-\eta})^{1-\mu}}{1-\mu} &= \frac{z_t^{\eta(1-\mu)}(\tilde{c}_t^{\eta} l^{1-\eta})^{1-\mu}}{1-\mu} \\
\sum_{t=0}^{\infty} \beta^t \frac{(c_t^{\eta} l^{1-\eta})^{1-\mu}}{1-\mu} &= \sum_{t=0}^{\infty} \beta^t \frac{z_t^{\eta(1-\mu)}(\tilde{c}_t^{\eta} l^{1-\eta})^{1-\mu}}{1-\mu} \\
&= \sum_{t=0}^{\infty} \beta^t \frac{z_t^{\eta(1-\mu)}(\tilde{c}_t^{\eta} l^{1-\eta})^{1-\mu}}{1-\mu} \\
&= z_0^{\eta(1-\mu)} \sum_{t=0}^{\infty} \tilde{\beta}^t \frac{(\tilde{c}_t^{\eta} l^{1-\eta})^{1-\mu}}{1-\mu}
\end{aligned}$$

where $\tilde{\beta} \equiv \beta(1+g)^{\eta(1-\mu)}$ and $\chi \equiv Tr_t/z_t$. The household budget constraint is detrended as

$$c_t + a_{t+1} = (1+r_t)a_t + W_t(1-l_t)\epsilon_t + Tr_t \tag{38}$$

$$\frac{c_t}{z_t} + \frac{a_{t+1}}{z_{t+1}}\frac{z_{t+1}}{z_t} = (1+r_t)\frac{a_t}{z_t} + \frac{W_t}{z_t}(1-l_t)\epsilon_t + \frac{Tr_t}{z_t} \tag{39}$$

$$\tilde{c}_t + (1+g)\tilde{a}_{t+1} = (1+r_t)\tilde{a}_t + \tilde{W}_t(1-l_t)\epsilon_t + \chi \tag{40}$$

Let $z_0^{\eta(1-\mu)} = 1$. Then the optimization problem of households can be written as

$$V_t(\tilde{a},\epsilon) = \max_{\tilde{c},\tilde{a}',l} \left\{ \frac{(\tilde{c}^{\eta} l^{1-\eta})^{1-\mu}}{1-\mu} + \tilde{\beta} E[V_{t+1}(\tilde{a}',\epsilon')] \right\} \tag{41}$$

$$\text{s.t.} \quad \tilde{c} + (1+g)\tilde{a}' = (1+r_t)\tilde{a} + \tilde{W}_t(1-l)\epsilon + \chi \tag{42}$$

The de-trended version of the firm's first order conditions is

$$W = z(1-\alpha)\left(\frac{K}{zL}\right)^{\alpha-1} \tag{43}$$

$$\tilde{W} \equiv \frac{W}{z} = (1-\alpha)\left(\frac{\tilde{K}}{L}\right)^{\alpha-1} \tag{44}$$

$$r = \alpha\left(\frac{\tilde{K}}{L}\right)^{\alpha} - \delta. \tag{45}$$

The asset market clearing condition is

$$\tilde{K} + \tilde{B} = \tilde{A}. \tag{46}$$

The government budget constraint is

$$\frac{G}{z} + (1+r)\frac{B}{z} = \frac{B'}{z'}\frac{z'}{z} + \frac{tax}{z} \tag{47}$$

$$\tilde{G} + (1+r)\tilde{B} = \tilde{B}'(1+g) + t\tilde{a}x. \tag{48}$$

Then we can define the government debt to GDP ratio as

$$b \equiv \frac{B}{Y} = \frac{B/z}{Y/z} = \frac{\tilde{B}}{\tilde{Y}}. \tag{49}$$

A.2 Numerical algorithm

Throughout this paper, the asset state space is discretized as follow. First, we set the upper bound of the asset grid $\overline{a}$ so that the measure of people whose asset holdings are above $\overline{a}$ is negligible. The lower bound of the asset grid is given by the borrowing constraint $\underline{a}$. Then we create an evenly spaced grid over $[\underline{a}, \overline{a}]$ with N_a points. In our calibration, we set $\overline{a} = 40$ and $N_a = 101$.

Stationary equilibrium

To compute the stationary distribution, we use the following procedure.

1. Make an initial guess of (r, w). Call it (r_i, w_i) with $i = 0$.
2. In iteration i, compute the absolute difference between supply and demand of capital and labor as follows.
   - Given (r_i, w_i), solve the dynamic programming problem of households and obtain the policy function $a'_i(a, \epsilon)$.
   - Compute the stationary distribution over asset holdings and labor productivity, $\mu_i(a, \epsilon)$, using $a'_i(a, \epsilon)$.
   - Once we have (r_i, w_i), $a'_i(a, \epsilon)$, and $\mu_i(a, \epsilon)$, we can compute the demand and supply for asset and labor.
3. If $|demand - supply| < \varepsilon$, change the guess of the prices and go to step 2.

When we compute the stationary distribution, we use the following procedure.

- Define a finer asset grid that has $N_a \times M$ points. We interpolate the policy function on this grid. We used $M = 3$ in our computations.
- When $a'(a, z) \in (a_i, a_{i+1})$, define

  $$p(a, z) = \frac{a_{i+1} - a'(a, z)}{a_{i+1} - a_i}. \tag{50}$$

  Then we contract a transition matrix of a' so that $a' = a_{i+1}$ with probability $p(a, z)$ and $a' = a_i$ with probability $1 - p(a, z)$. The stationary distribution is obtained by using the eigenvector method (Badshah, Beaumont, and Srivastava (2013)). That is, although the stationary distribution is the unique eigenvector associated with eigenvalue 1, there are many other eigenvalues which is very close to 1 which is hard to distinguish from exact 1 computationally. To avoid this problem, we add a very small number to the transition matrix and use the eigenvector associated with 1 as a stationary distribution.

Transition path

We follow Conesa and Krueger (1999) to make an initial guess of capital and labor and keep iterating on it until we reach convergence.

1. Set the initial and terminal conditions. Assume that the economy reaches the new stationary equilibrium at period T.

2. Make an initial guess on the sequence of aggregate labor and detrended aggregate capital $(L_t^i, \tilde{K}_t^i)_{t=0}^T$ with $i = 0$. Using this information we can compute the prices $(r_t^i, \tilde{W}_t^i)_{t=0}^T$ from the first order conditions of the firms.

3. Given $(r_t^i, \tilde{W}_t^i)_{t=0}^T$, obtain the updated sequence $(L_t^{i*}, \tilde{K}_t^{i*})_{t=0}^T$ as follows.

    (a) Given the prices and the terminal condition $V_T(a, \epsilon) = V^{SS}(a, \epsilon)$, solve the households' problem in a backward manner.

    (b) Given the policy function, update the distribution of households forward from $\mu_0(a, \epsilon) = \mu^{SS}(a, \epsilon)$.

    (c) Once we know the policy function and distributions, we can compute the aggregate capital and labor.

4. Check if $\max\{|\tilde{K}_t^i - \tilde{K}_t^{i*}|, |L_t^i - L_t^{i*}|\} < \varepsilon$. If not, update the sequence of detrended capital and labor by the weighted average of the initial guess and updated sequence, $K_t^{i+1} = \omega \tilde{K}_t^i + (1-\omega)\tilde{K}_t^{i*}$ and $L_t^{i+1} = \omega L_t^i + (1-\omega) L_t^{i*}$, and go back to step 2. If the condition is satisfied, done.

There is no guarantee that through this process the sequence $(K_t^i, L_t^i)_{t=0}^T$ converges to the equilibrium transition dynamics as i grows. We tried different values of the weight on the old guess, ω, and then set $\omega = 0.9$ and obtained convergence.