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A Proposal for Lifetime Income Taxation: The Vickrey Mechanism Revisited

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Abstract

This paper proposes a new scheme for personal income taxation that taxes lifetime, not annual income, through a reformulation of Vickrey's income averaging. Lifetime income taxation contributes to redistribution according to lifelong ability to pay while ensuring horizontal equity among taxpayers with equal lifetime income, irrespective of income generation patterns. It can also play an insurance role, as high income in one year will be leveled with a decrease in income (or deficit) in the event of a disaster. Moreover, unlike annual income taxation, there will be no incentive to delay the "realization" of income, so lifetime income taxation is neutral with regard to type of income, such as capital gains, stock options, and retirement allowances.

Keywords: Lifetime income taxation, horizontal equity, progressive taxation, insurance, tax deferral

JEL Classification: H20, H24

1. Introduction

The spread of COVID-19 has revealed deficiencies in existing safety nets. The ILO (2021) estimates that global labor income (before income support measures) in 2020 declined by 8.3% to $3.7 trillion or 4.4% of global GDP. The OECD (2020) reports that the economic consequences of the COVID-19 pandemic have fallen unevenly on workers, with lower paid and part-time workers and the self-employed being more exposed to income losses. In the United Kingdom, 75% of the self-employed reported having experienced a drop in earnings, compared to less than 25% of salaried workers. In normal times, they would have been independent and supported the economy. Therefore, without adequate and effective support for them in times of emergency, the economy cannot be expected to recover once the crisis is over. The government has sought to expand assistance for sole proprietors, but this has remained ad hoc. On the other hand, the economic and social environment is changing drastically with the progress of globalization and digitalization.

There has been an increase in the number of workers with unstable income, such as freelance and gig workers. Along with redistribution to reduce the income gap, the role of income tax as insurance has become more important than ever. A new tax system will be required and needs to be put in place for this new economic and social environment. In the case of corporate tax, companies are permitted to carry forward or carry back losses on the premise of a going concern that corporate activities will continue into the future. Corporate tax can thus be levied at a flat rate on the present value of future profits.

This paper proposes to introduce a similar mechanism for personal income tax, changing annual income taxation to lifetime income taxation while preserving progressivity. Note that while the form of taxation proposed in this paper is different from basic income, which ensures the same income for all, it nonetheless serves as insurance to stabilize disposable income. In social insurance programs, such as unemployment and pension benefits, a history of previous earnings is reflected, and in this regard, these benefits are based on lifetime earnings. I aim to incorporate the lifetime perspective into personal income tax. In the present personal income tax system, the ability to pay is measured according to annual income. However, taxpayers who have earned a high income in the current year due to the listing of stocks and/or the realization of capital gains will not necessarily continue to gain high income in subsequent years. In economics, it is an individual's welfare level that determines their ability to pay, which depends on their lifetime income. Although value-added tax (VAT) is criticized for being regressive from an annual income perspective, on a lifetime basis, it is a tax equivalent to lifetime income taxation so that the economic effect is the same. Therefore, the lifetime incidence of VAT can be measured quite differently from annual VAT (Casperson and Meltdalf 1994). VAT is known to be advantageous in raising revenue, but it has a limited redistribution function. A progressive tax on lifetime income, on the other hand, can also serve as a progressive consumption tax from a lifetime perspective.

Related to lifetime income taxation, the averaging of income for tax purposes was introduced in the United States from 1964 to 1986 and Canada from 1972 to 1988; the averaging was not over a full lifespan but limited to a certain number of years. The formula for averaging personal incomes in Canada is contained in Davies (1975).[1]Because of the administrative burden of managing income information, it was abolished to simplify the tax system. However, with the recent advances in digital technology, lifetime

[1] In 1949, after World War II, the Shoup Mission proposed a fundamental tax reform for Japan and recommended a special provision for fluctuating income, such as capital gains, by averaging the tax burden over several years (Shoup1949); however, income averaging was never fully implemented. Vickrey was a member of the Shoup mission and was in charge of personal income tax reform.

income taxation has become possible without compromising simplicity. Jacobs (2017) discusses the possibility of lifelong income taxation in the light of the development of digital technology. As a part of renovating the Canadian income tax system, Boadway (2019) proposes that general income averaging be re-instituted given that the Canadian tax-transfer system has become less effective in mitigating market-induced income volatility. A potential objection may be that the value of income averaging is not great given that personal income taxes have been less progressive with smaller differences in marginal tax rates. Progressive taxation may have been constrained, however, by considerations that annual income taxation imposes higher burdens on taxpayers with lumpy and fluctuating incomes. Indeed, there is empirical evidence that household income has become increasingly volatile. Moffitt and Zhang (2018) review major papers that study income volatility using PSID in the United States. Gordon and Wen (2017) provide an extensive survey of the experience of averaging income taxation and examine its effects. Their findings are that while welfare gains from averaging is generally low, it can be high for high-income earners. Batchelder (2003) proposes income averaging that targets low-income families by averaging the earned income tax credit (EITC) over a two-year period, given that they tend to experience higher income volatility than middle- and high-income households.

Vickrey (1939) proposed the averaging of income for tax purposes. My proposal is based on his pioneering work. Income tax averaging and lifetime income tax differ, however, in their treatment of normal returns on savings. As noted above, the latter is effectively a tax on consumption, whereas the former can encompass comprehensive income taxation. In addition, income averaging applies to a certain number of tax years, five years for instance, and thus is shorter than a lifetime. Steinerberger and Tsyvinski (2020) have formulated axioms of income tax averaging. They account for general weights on income, whereas the present study calculates lifetime income using an arithmetic average. Lifetime income taxation may be supported from the viewpoint of optimal taxation theory, in that it is desirable to utilize past income information to deal with informational asymmetry (Golosov et al. 2011). Erosa and Gervais (2002) modeled optimal taxation in a standard overlapping generation setting with both wage and capital taxes being levied on an annual basis while allowing for age dependence. I propose that lifetime income tax can enhance the insurance function. In a related vein, Varian (1980) examined optimal income tax as insurance in the presence of income uncertainty. His work has been extended by Chetty and Saez (2008), who have incorporated the private insurance market. Stepner (2019) finds evidence that the progressive shape of taxes and transfers (on an annual income basis) provides the majority of social insurance against such risks as layoffs and hospitalization using the Canadian tax record. Bovenberg and

Sorensen (2006) studied optimal lifetime income taxation and social insurance in which compensation for the loss of earning capacity depends on previous labor income. Lifetime taxation can serve to remove the incentive of tax deferral. Auerbach (1991) and Auerbach and Bradford (2004) have proposed capital income taxation that can be neutral with respect to the realization of capital gains. Their tax structure is flat, although the tax rates can be dependent on holding periods, whereas the present paper considers progressive taxation.

The remainder of this paper is organized as follows. Section 2 addresses deficiencies in current year income taxation. A lifetime income tax is proposed in Section 3, assuming a zero-interest rate for simplicity. A more general form of lifetime income tax is presented in Section 4. Section 5 provides some concluding remarks. The formal model is given in the Appendix to consider the incentive effect of lifetime income taxation.

2. Deficiencies of current year income tax

In this section, I address the deficiencies of annual income taxation using simple numerical examples. An interest rate of zero is assumed for simplification. The three taxpayers, Mr. A, Mr. B, and Mr. C in Table 1 all have equal lifetime incomes of $300,000 over five periods. However, their income generation patterns are different. In the present context, taxable income includes temporary income, such as retirement allowance and capital gains, in addition to salary and business income. (The taxable income may become negative due to capital losses and losses due to disasters.) While Mr. A's earnings are constant at $60,000, there are variations for Mr. B, and Mr. C gains a high temporary income of $180,000 only in the fourth period. In the case of the annual income tax being applied, there will be a difference in income tax, even though the lifetime income is the same. In the example, we applied the tax table provided in Appendix 1: the income tax is at a marginal tax rate of 10% on a taxable income up to $30,000, at 20% on an income of between $30,000 and $60,000, and at 35% on an income of over $60,000 (Appendix 1). For zero income, the tax amount is assumed to be zero. There is no change in the essence of the discussion when benefits for low income, such as a negative income tax, are incorporated.

In this example, Mr. A's total income tax liability over the five periods is $45,000, whereas Mr. B's income tax increased to $58,500, and Mr. C is taxed even higher at $63,000. Consequently, horizontal inequity arises among taxpayers on a lifetime basis. Even the vertical inequity may be undermined if a taxpayer with a lower lifetime income but a volatile income bears a higher lifetime tax liability than say Mr. A. in the example.

Table 1: Annual income taxation

	Mr.A		Mr.B		Mr.C	
Period	Taxable income	Annual income tax	Taxable income	Annual income tax	Taxable income	Annual income tax
1	60,000	9,000	30,000	3,000	30,000	3,000
2	60,000	9,000	90,000	19,500	30,000	3,000
3	60,000	9,000	30,000	3,000	30,000	3,000
4	60,000	9,000	120,000	30,000	180,000	51,000
5	60,000	9,000	30,000	3,000	30,000	3,000
Sum	300,000	45,000	300,000	58,500	300,000	63,000

Note: Tax amount = 0.3 x income up to \$30,000, tax amount = 0.2 x (income – \$30,000) + 0.1 x \$30,000 for over \$30,000 and less than \$60,000, tax amount = 0.35 x (income for over \$60,000 – \$60,000) +0.2 x (\$60,000 – \$30,000) +0.1 x \$30,000

Annual income tax does not adequately fulfill the insurance function. Suppose Mr. B's income is completely lost due to a natural disaster like COVID-19 in the 4th and 5th periods. Accordingly, the income tax liability becomes zero. However, as opposed to corporate tax, there are no tax refunds such as carry backs or future tax cuts as loss carryforwards. Of course, there will be savings as self-support, that is, self-insurance, but it is not always possible to anticipate a disaster in advance. In the absence of a disaster, savings for self-insurance could turn out to be excessive. While the safety net guarantees a minimum standard of living, the function of insurance is to maintain a standard of living close to income during normal times, that is, to equalize income during normal times and emergencies, which is missing in the current year income taxation scheme.

In addition, progressive taxation of annual income induces a delay in income realization. This is because income taxation is not incurred on an accrual basis, but is levied when it is realized.[2] For example, when Mr. B has a high income, he can reduce his income tax liability by deferring \$30,000 out of the \$90,000 income in the second period to the third period when his income is low, and so it is at the marginal tax rate. The tax burden is averaged at the discretion of the taxpayer, but it is horizontally unfair between taxpayers who have such an opportunity and those who do not. Furthermore, the lock-in effect of retaining unrealized gains on stocks may impair the liquidity of investment funds and cause inefficiencies in capital markets. On the other hand, if the progressive structure is relaxed to suppress tax avoidance (arbitrage), the income tax redistribution function will be impaired.

Table 2: Deficiencies of annual income tax

[2] Slemrod (2001) extends standard leisure and consumption decisions to incorporate both labor supply and tax avoidance behaviors. Empirically, elasticities of taxable income serve as sufficient statistics to estimate these responses. Saez et al (2012), in a review of the literature, conclude the best available estimates range from 0.12 to 0.4, with a medium of 0.25.

Period	Mr.B					
	Taxable income		Annual income tax		Taxable income	Annual income tax
	Normal	Emergency	Normal	Emergency	Realization basis	
1	30,000	30,000	3,000	3,000	30,000	3,000
2	90,000	90,000	19,500	19,500	60,000	9,000
3	30,000	30,000	3,000	3,000	60,000	9,000
4	120,000	0	30,000	0	75,000	14,250
5	30,000	0	3,000	0	75,000	14,250
Sum	300,000	150,000	58,500	25,500	300,000	49,500

Note: The same tax structure as Table 1 is assumed.

3. The Case for lifetime income taxation

Now, I present the lifetime income taxation recommended in this paper. Of course, lifetime income is not known in advance, therefore, the accumulated amount of income realized by the taxpayer from the beginning (e.g., job carrier) to the present is used. If using the "absolute level" of lifetime income, progressive taxation is more likely to occur in later years of life when income is being accumulated. To avoid this, I adopted an "average" income and applied a similar tax structure as the current progressive tax. This is a lifetime-based income-leveling tax. Progressive tax is levied on the cumulative average of income, and this is multiplied by the period up to the present. As a result, the total amount of income tax that would have been levied if the average amount earned from the beginning to the present is calculated. In addition, the total amount of income tax up to the previous period (calculated in the same way) is deducted so that taxation does not accumulate. This corresponds to the refunding of the income tax paid in the previous period. In other words, taxation and refunding according to the cumulative average income are repeated. This is ruminant of Vickrey (1939). [3]

In this section I continue to assume a zero-interest rate. Specifically, letting J be the current period and T (・) the tax function, the taxable amount for this period (= period J) is given as follows:[4]

$$\tau_J = J * T\left(\frac{1}{J}\sum_{j=1}^{J} z_j\right) - (J-1) * T\left(\frac{1}{J-1}\sum_{j=1}^{J-1} z_j\right) \quad (1)$$

where τ_J denotes tax liability in period J, and z_j is real income that is realized in period j. z_j may take negative values if there is capital loss or income falls below the taxation thresholds. The taxable amount for this period can be expressed as:

$$\tau_J = T(\bar{z}_{J-1}) + J * [T(\bar{z}_J) - T(\bar{z}_{J-1})] \quad (1'),$$

where

[3] Vickrey's original proposal is illustrated in Appendix 4. A main difference between Vickrey(1939) and the current proposal is treatment of normal return which is abstracted in this section.

[4] The period J depends on the taxpayer's age. For taxpayers who have been working since 2000, 2021 will be J = 21, while if the start of employment is from 2015, it will be J = 6. In that sense, it is similar to "age-dependent income tax."

$$\bar{z}_J \equiv \frac{1}{J}\sum_{j=1}^{J} z_j = \frac{1}{J} z_J + \frac{J-1}{J}\bar{z}_{J-1} \quad \text{and} \qquad \bar{z}_{J-1} \equiv \frac{1}{J-1}\sum_{j=1}^{J-1} z_j$$

Then it can be interpreted that the amount that has not been paid (not refunded) by the current period (i.e., period J) is added (reduced) to the tax amount corresponding to the accumulated average up to the previous period. [5]Any tax preference or special provisions that reduce tax liability in period J, say by $J \times \Delta T(\bar{z}_J)$, is offset by larger τ_{J+1} as the refunding in period J+1 is reduced by the same amount. Note also that lifetime income taxation can also serve to simplify the income tax system since special treatments for lumpy income such as retirement benefits and capital gains are not required (Vickrey, 1969). When income remains constant during these periods, the cumulative average matches the annual income. Therefore, the taxable amount for each period is the same as the annual income tax. If the tax function is flat, the lifetime income tax is equivalent to annual income tax as well.

Now take the case of Mr. B, whose income fluctuated greatly over the periods. The tax function is assumed to be the same as in Table 1. His income for the second period is \$90,000, but the cumulative average of \$60,000 up to this point (= (\$30,000 + \$90,000) ÷ 2) is taxable under lifetime income taxation. Applying the same tax function as the current year's income tax, the tax amount will be \$9,000. If this cumulative average were to occur equally over the two periods, the total income tax would be \$2 x \$9,000 = \$18,000. With the \$3,000 already paid in the previous period being refunded, the final tax amount for the second period becomes \$15,000 (= \$2 x \$9000 – \$3,000). After the third period, the taxation and refunding of the cumulative average income is repeated in the same manner. The cumulative average for the final period is equal to the average lifetime income of \$60,000 (= \$300,000 ÷ 5 periods). As a result, the total income tax liability over the five periods is \$45,000 (= \$ 5 × T (\$60,000)), which is the same as the tax burden of Mr. A. The same applies to Mr. C’s case, and his lifetime tax amount also amounts to \$45,000.

Table 2: Lifetime income taxation

	B氏			
Period	Taxable income	Accumlated average	Lifetime income taxation	
			T(・)	Tax amount
1	30,000	30,000	3,000	3,000
2	90,000	60,000	9,000	15,000
3	30,000	50,000	7,000	3,000
4	120,000	67,500	11,625	25,500
5	30,000	60,000	9,000	−1,500
Sum	300,000			45,000

[5] Conventional annual income taxation may still serve as a withholding tax so it can be supplementary to tax enforcement of lifetime income taxation.

Insurance Function			
Taxable income	Accumlated average	Lifetime income taxation	
		T(・)	Tax amount
30,000	30,000	3,000	3,000
90,000	60,000	9,000	15,000
30,000	50,000	7,000	3,000
0	37,500	4,500	−3,000
0	30,000	3,000	−3,000
150,000			15,000

Note: The same tax structure as Table 1 is assumed.

There will be some debate about the period over which the cumulative average is taken (five periods in the above example). Of course, it does not include childhood (supported by parents), and individuals who work but depend on the primary earner of the household, such as university students and part-time housewives, should not be subjected to lifetime income tax. Their income will then continue to be taxed on a calendar year basis. In addition, if the employee returns to work after a certain period of retirement, the cumulative average may be taken again with the timing of the return to work as the initial period.

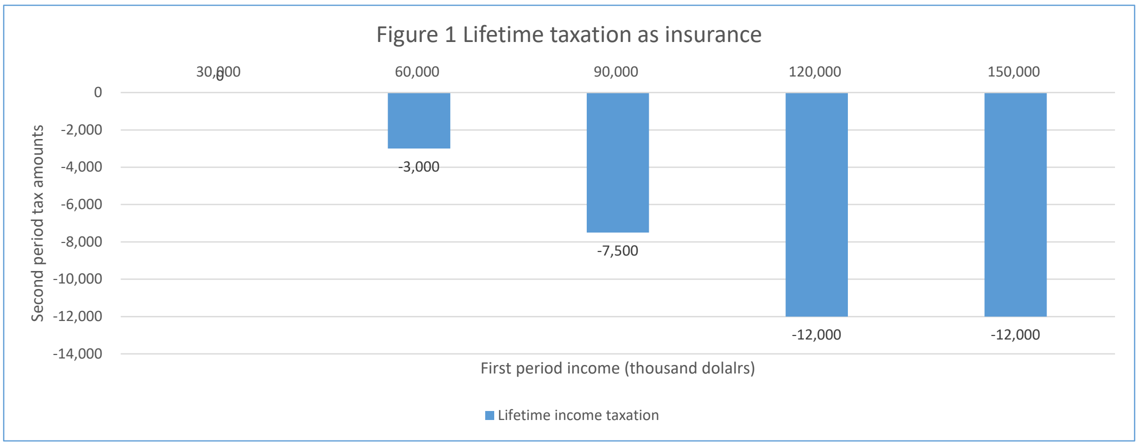
Now let me address an insurance function of lifetime income taxation, leaving the incentive effect of tax deferral to the next section. As shown in Table 2, we assume that Mr. B's income has been lost due to an event such as a disaster after the 4th period. In contrast to the annual income taxation, a refund of $3,000 will occur because the cumulative average drops significantly from the previous period. This refunded amount increases as the income level in the normal period increases. Figure 1 compares the income in normal times (1st period) and the refunded amount in the 2nd period, with the income at the time of disaster (2nd period) as zero. All the taxpayers in Figure 1 have no income in the second period, but the tax treatment differs depending on their income in the previous period. Here, with the taxable income in the normal period being $60,000, the net refund amount is $3,000 in the period of the emergency, while if it is over $120,000, the amount of $12,000 is transferred. One could say that such treatment is unfair, but these taxpayers are not equal on a lifetime basis. The standard of living in the normal period is subsequently maintained. In general, this can be confirmed by

$$\frac{d}{d\bar{z}_{J-1}}\tau_J = (J-1) * \left(T'(\bar{z}_J) - T'(\bar{z}_{J-1})\right) \leq 0 \qquad (2)$$

if and only if $\bar{z}_J \leq \bar{z}_{J-1}$, so in the case that the current income declines relative to the previous average income, the tax liability in the current period decreases. Moreover, because the tax function is progressive and convex,

$$T(z_J) > J * T\left(\frac{1}{J}z_J + \left(1 - \frac{1}{J}\right)\bar{z}_{J-1}\right) - (J-1) * T(\bar{z}_{J-1}) = \tau_J \quad (3)^6,$$

Tax liability becomes negative and thus, net credit is provided when z_J = 0 and T(0) = 0.



Note: The same tax structure as Table 1 is assumed.

At this point, some additional remarks need to be made. First, in calculating the cumulative average income, households that are exempt from tax under the current system are also required to file a tax return to report income. There may be criticism that declaring a small amount of income may impair the convenience of taxpayers. However, even in the implementation of a negative income tax, capturing the accurate income of low-income earners is required. It is noteworthy that lifetime income tax in this study can be integrated with the benefits to low-income earners alongside the refunding to taxpayers whose income has decreased from the previous year. Second, in the case of lifetime income taxation, the government tax revenue in each period will decline relative to the current tax system, as can be seen in Eq. (3).[7] If we assume that income is independently distributed for simplicity, the law of large number applies. Then the expected or average revenue loss compared to annual income taxation is approximated by

$$E[T(z_j)] - E[T(\bar{z}_J)] \approx \frac{1}{2} T''(\bar{z}_J) \times E[(z_j - \bar{z}_J)^2] \qquad (4)$$

The revenue loss is larger as the tax structure is more progressive and the variance is larger However, since the incentive for tax avoidance is suppressed, it will be easier to maintain a progressive structure and secure tax revenue than in the case of annual income taxation. Indeed, given that taxpayers are risk averse, they will be willing to pay a risk premium to cover disaster-led income risk in the form of a higher tax in normal times.

[6] Eq. (3) holds with interest rate as discussed in section 4 as well.

[7] By Taylor expansion, $T(z_j) \approx T(\bar{z}_J) + T'(\bar{z}_J) \times (z_j - \bar{z}_J) + \frac{1}{2} T''(\bar{z}_J) \times (z_j - \bar{z}_J)^2$

Appendix 3 shows that shifting from annual to lifetime income taxation in a tax revenue neutral manner can be welfare enhancing. With widening disparities caused by financial income, such as capital gains, there has been a political requirement to strengthen progressive taxation. Lifetime income taxation can correspond to it effectively and allow more room for the redistribution function to be exerted.

4. General case of lifetime income taxation.

Thus far, we have assumed a zero-interest rate. Once the interest rate is accounted for, the lifetime income of the household is expressed in terms of the discounted present value. Here, the discounted present value is from the initial perspective. Accordingly, the income in the period closer to the present is largely discounted. The interest rate used for the discount rate (and normal return, as described later) may be set as the average of government bond interest rates. Under lifetime income taxation, the present value of taxable income up to each period is cumulatively averaged. [8] The tax function T (・) applies to this cumulative average. Then, the taxation of the cumulative average up to the present period and the refunding of the taxation in the previous period are repeated. The taxable amount in each period is translated at the interest rate (the same as the discount rate) to the current value. That is, with J = 1 being the initial period for calculating the lifetime income,

$$\tau_J = (1+r)^{J-1}\left\{J * T\left(\frac{1}{J}\frac{z_J}{(1+r)^{J-1}} + \frac{J-1}{J}\bar{z}_{J-1}\right) - (J-1) * T\left(\bar{z}_{J-1}\right)\right\} \qquad (5)$$ [9]

where r is the real interest rate, which is assumed to be constant for expositional convenience and

$$\bar{z}_{J-1} \equiv \frac{1}{J-1}\sum_{j=1}^{J-1}\frac{z_j}{(1+r)^{j-1}}$$

When the tax amounts from the initial period to the current period are summed up and

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[8] When valued from the first period perspective, inflation affects both taxable income and the discount rate in the same manner, so the real value is kept constant. To see it, write price level in period j by P_j with $P_1 = 1$, and let i and π respectively nominal interest and inflation rates. Then we have:

$$\frac{P_j z_j}{(1+i)^j} = \frac{(1+\pi)^j z_j}{(1+\pi)^j (1+i)^j} = \frac{z_j}{(1+r)^j}$$

In practice, nominal interest rate is subject to monetary policy. If monetary policy is tied to inflation, for instance, following the Taylor rule, the above argument will hold. In Vickrey (1939), on the other hand, the current value of the averaging income is accounted for. I replicated Vickrey's (1939) calculation in Appendix 2.

[9] In general, net tax payment in period J can be $\tau_J - b_J$ subtracting a fixed amount of tax credits b_J from (5). If the tax structure is flat at a tax rate of t with T(0)=0, (5) can reduce to annual income tax: $\tau_J = t \times z_J - b_J$.

discounted to the initial value, the total tax liability matches

$$\sum_{J=1}^{J} \frac{\tau_J}{(1+r)^{J-1}} = J * T\left(\frac{1}{J}\sum_{j=1}^{J} \frac{z_j}{(1+r)^{j-1}}\right) \tag{6}$$

As a result, taxation on lifetime income will be realized regardless of the generation of income in each person's life cycle and working and survival periods. This fulfills Vickrey's requirement that "the discounted value of the series of tax payments made by any taxpayer should be independent of the way in which his income is allocated to the various income years" (p. 382).

Table 3: Lifetime income taxation

Period	Taxable income		Accumulated average	Annual income taxation
	Current value	Present value		Current value
1	40,000	40,000	40,000	5,000
2	61,200	60,000	50,000	9,420
3	41,616	40,000	46,667	5,323
4	106,121	100,000	60,000	25,142
5	64,946	60,000	60,000	10,731
Sum		300,000		

Life time income taxation			Incentive of tax deferral		
T(･)	Tax amounts		Realized income		Accumulated average
	Current value	Present value	Current value	Present value	
5,000	5,000	5,000	40,000	40,000	40,000
7,000	9,180	9,000	51,000	50,000	45,000
6,333	5,202	5,000	52,020	50,000	46,667
9,000	18,041	17,000	84,897	80,000	55,000
9,000	9,742	9,000	86,595	80,000	60,000
		45,000		300,000	

Note: The interest rate is assumed to be 2%.

Note 2: The same tax structure as Table 1 is assumed.

Consider again the tax function shown in Table 1. In the present context, the threshold of the marginal tax rate is interpreted in terms of the present value. For example, the tax amount for the cumulative average of \$46,667 up to the third period at the present value is \$6,333. Therefore, the tax amount, if the income remains constant at the cumulative average until the third period, is \$19,000 (= 3 × \$6,333). As shown in Table 3, multiplying the cumulative average taxable amount of \$7,000 in the previous term by two periods and deducting it, the net tax liability in the third period is \$5,000 (= 3 × \$6,333-2 × \$7,000). Translating it at an interest rate of 2% to the value of the third period, the taxable amount in this period becomes equal to \$5,202 (=$(1+0.02)^2$× \$5,000). Given that the discounted present value of lifetime income is \$300,000, the PV of lifetime income taxation is \$45,000. As long as the lifetime income is \$300,000, the amount of the present value of lifetime income tax remains unchanged at \$45,000, irrespective of the pattern of income generation.

Under the lifetime income taxation, tax deferrals are irrelevant. The last two columns of Table 3 illustrate how the accumulated average changes when the realization of income is deferred. To be concrete, \$1,000 in the present value (so \$1,020 in the current value) is deferred from period 2 to the next period. Another realization of \$2,000 is delayed from period 4 to the last period. This does not alter the average life income in the end. The intuition is the following. Suppose that deferred taxation could reduce the tax amount by \$1,000 in this period. According to Eq. (5), the tax amount for the next time will increase because of the reduction in refunding for the previous tax by \$1,020 (=\$1,000×1.02)) with 2% interest rate. In terms of the present value, this offsets the tax savings. Indeed Eq. (6) implies that the tax burden in the present value is independent of patterns of realizing income up to period J given an accumulated average of \$6,000. To see the point more clearly, consider that income M_J deferred from period J to the next period is saved at an interest rate of r. Such a saving may be retained earnings of the company and it is paid with interest in the future to the company's own employee. This presumption also implies that there is no economic gain from deferring tax. Given that y_j is accrued income in period j

$$z_j = y_j + (1+r)M_{j-1} - M_j \qquad (7)$$

The above may be then added with 1+r being multiplied up to period J

$$M_J = \sum_{j=1}^{J}(1+r)^{J-j}(y_j - z_j) \quad \text{or} \quad \sum_{j=1}^{J}\frac{z_j}{(1+r)^{j-1}} = \sum_{j=1}^{J}\frac{y_j}{(1+r)^{j-1}} - \frac{M_J}{(1+r)^{J-1}} \qquad (7')$$

There is thus a one to one correspondence between the discounted present value of realized income and the tax deferral $M_J/(1+r)^{J-1}$. Then we have

$$\frac{d}{dM_J}\tau_J + \frac{1}{1+r}\frac{d}{dM_J}\tau_{J+1} = (1+r)^J\left\{-T\left(\frac{1}{J}\sum_{j=1}^{J}\frac{z_j}{(1+r)^{j-1}}\right) + T\left(\frac{1}{J}\sum_{j=1}^{J}\frac{z_j}{(1+r)^{j-1}}\right)\right\} = 0 \qquad (8)$$

Therefore, total tax liability remains the same, which implies that the tax deferral does not mitigate the lifetime tax burden. Appendix 2 establishes that M_J is redundant in the household's optimization once his saving decision is optimized.

Beside the tax deferral aiming to reduce tax liability, real behavioral responses also exist. The labor supply may respond to the taxation on the extensive (entry) margin and the intensive (hours worked) margin. The taxation may also affect the inter-temporal substitution of labor supply and human capital investments. Meghir and Phillips (2009) have provided an overview of the impact of tax and benefits on labor supply. Hours of work and labor participation for single mothers are relatively sensitive to taxes and benefits. In Appendix 2, I give a formal model of lifetime income taxation with labor hours as a real behavioral response. It can be shown that

$$-\frac{U_{L_J}}{w_J U_{c_J}} = 1 - \frac{E_J\lambda_N T_N'}{E_J\lambda_N} = 1 - E_J[T_N'] - Cov\left(\frac{\lambda_N}{E_J\lambda_N}, T_N'\right) \qquad (9)$$

where $T_N = T(\bar{y}_N)$ and *N* is the last period. The conditional expectation is taken on the future taxation since the model allows for uncertainty. The left-hand side refers to the marginal rate of the substitution between labor and consumption with w_J being the wage rate in period J. The lifetime taxation will encourage (discourage) labor supply relative to the annual taxation if $T'(\bar{y}_N) < (>) T'(y_J)$. Turning to the last term, it is plausible that the covariance term takes negative values given that the marginal utility of income is declining in income and the tax is progressive. The covariance term serves to mitigate the disincentive effect of the taxation. Eq. (9) can easily be extended to the choice of the extensive margin. Given that the tax wedge relies solely on the tax rate in the last period, the inter-temporal substitution of labor is not distorted. Relatedly Ham and Reilly (2013) have found large intertemporal labor supply elasticities. Using the tax holiday in Swiss cantons as natural experiment, on the other hand, Martínez et al. (2018) have found little evidence of a labor supply effect overall, but also noted shifts in earnings among the self-employed and high-income earners who can easily adjust their working hours.

In the present context, the distinction between normal return and excess return matters in the calculation of lifetime income. In principle, however, normal return, like interest income from savings, is derived from previous earnings such as wages and is not counted in lifetime income. Of the income generated from investment and savings, on the other hand, excess return should be included in lifetime income. Indeed, the Mirrlees Review overall recommends taxing excess returns while exempting normal returns, (Mirrlees et al. 2011).[10] For example, suppose that one bought stock for \$1,000 and sold it for \$11,000 in the next period. The capital gains would be \$1,000 (= \$11,000 – 10000). However, if one saves \$10,000 with an interest rate of 2%, \$200 (= 2% × \$10,000) would be a normal return. In this case, the excess return becomes \$800 (= \$1,000 – \$ 200). [11]This excess return is obtained by inflating the purchase amount (= \$10,000) at an interest rate of 2% (= (1 + 0.02) × \$ 10,000) and subtracting it from the sales amount (= \$11,000) in the current period.[12] Appendix 2 establishes that the decision of the risky investment made in period

[10] There are arguments in favor of taxing normal return inclusive capital income. For instance, if high-skilled workers save more, taxing capital income can serve to supplement labor income taxation from an optimal tax perspective (Saez 2002). For the thread in the literature, see Bastani and Waldenström (2020). With a flat tax on interest at a rate of t, the discount rate will be replaced by r(1-t). This will give rise to the lock in effect, however, that is discussed in Appendix 4.

[11] An excess return on real assets such as housings may be able to be calculated likewise taking the difference between imputed income and market or mortgage interest rate.

[12]Such excess profit taxation is "tax equivalent" to cash flow taxation. Suppose that one purchase an asset at P_J in period J and sell it at the price of P_{J+1} in the next period. We have in the present value:

$$\frac{(P_{J+1} - P_J) - r * P_J}{1+r} = \frac{P_{J+1}}{1+r} - P_J$$

Therefore, the excess return taxation can be transformed into cash flow tax with the purchase of a risky asset being immediately deductible.

J is fulfilled

$$E_J\left[(\rho_{J+1}-r)\lambda_{J+1}\left(1-\frac{E_{J+1}\lambda_N T_N'}{E_{J+1}\lambda_N}\right)\right]=0 \qquad (10)$$

where ρ_{J+1} is the rate of return that is uncertain ex ante and λ_{J+1} is the marginal utility of income in period J+1. Again, only the marginal tax rate in the last period appears in the first order condition. Note that if T_N' is constant, Eq. (10) reduces to the Domar and Musgrave (1944) result implying that the tax may stimulate risk taking.

As in the case of the Norwegian personal tax system as illustrated by Sorensen (2005), the imputed return, the rate-of-return-allowance that is deducted from taxable shareholder income, may apply to calculate the excess return or equivalently the cash flow. In the case that return on investment falls below r, the loss may be carried over with interest. Concerning business assets, rent taxation can be achieved by transforming the tax base into cash flow base with receipts being taxable, whereas expenses including real investment had been deductible as was originally proposed by the Meade report (Meade, 1978) and updated by the Mirrlees Review (Mirrlees et al. 2011). For closely held assets where it is difficult to assess initial value, there will be a need for some presumptive methods to calculate excess return. It would be also possible to expand tax-exempt saving accounts so that contributions are tax deductible and interest income from such savings is tax exempt, and tax is levied on withdrawals—this is known as registered assets in the proposal of expenditure tax.

In some OECD countries such as France and Germany, there is a cumulative taxation that integrates the gift tax and inheritance (Drometer et al. 2018). For example, in the case of Germany, the amount of property transfer over the past 10 years from the period of inheritance is accumulated and taxed. This serves to neutralize the timing of transferring wealth to the next generations. In addition to wages and excess profits, heritage and gifts also make up lifetime income. Lifetime income tax can thus encompass inheritance and gift taxes. However, there are many inherited assets that are difficult to value, including unlisted stocks and land. In this case, the taxable income is set equal to the acquisition value, and later, when these asses are sold, the excess return can be taxed. If the asset is initially valued low, the normal return as calculated by the market interest rate will be low as well, which in turn raises the excess return. There may arise a case in which tax is not payable due to inadequate liquidity. If so, tax payment can be delayed by, for example, $\Delta T(\bar{z}_1)$ in period 1, which in turn lowers the future refund and thus increases tax liability: when the tax payment is postponed to the next period, the refund declines by $(1+r)\Delta T(\bar{z}_2)$ and τ_2 increases accordingly. As a result, lifetime tax payment in the discounted present value does not change. As such, the lifetime taxation serves to mitigate the liquidity problem, for it can allow for some flexibility regarding the timing of the taxation overtimes for a taxpayer's convenience, subject to tax liabilities being cleared by a certain period, which needs to meet Eq. (6).

In the transition from calendar year income taxation, there arises the issue of whether to include the assets held by individuals in lifetime income. If these assets originated from pre-transition income, they could be double taxed because of the annual income taxation at that time. In order to apply lifetime income tax strictly, it is necessary to re-calculate the cumulative average and refund past taxation, but this is difficult in practice. Rather, it is compromising, but it is plausible not to tax assets in the transitional period as lifetime income.[13] (As mentioned above, excess returns, including unlisted stocks, will be subject to lifetime income tax when they are realized in the future.)

Regarding administration and compliance, it should be noted that lifetime income taxation does not require too much information for enforcement compared to current year income taxation. The cumulative average up to the current period is given as a weighted average of the income (in the present value) of this period and the cumulative average up to the previous period. In addition, once the interest rate to be applied is determined, the present value of income in each period will also be set. Given that the final taxable amount of an individual will depend on the final tax system, and the income tax system is revised almost every year, there may be a high degree of uncertainty. However, even under the current tax system, individuals and companies face similar uncertainties when making long-term decisions, such as investments. The return on investment will occur in the future, and the corporate tax and income tax at that time are uncertain. Therefore, the uncertainty of future taxation is not limited to lifetime income taxation.

Conclusion

The advantages of the lifetime income taxation proposed in this study can be summarized as follows. First, it contributes to redistribution according to lifelong ability to pay. Second, horizontal equity is ensured among taxpayers with equal lifetime income, irrespective of income generation patterns. Third, high income in one year will be leveled with the decrease in income (or deficit) in the event of a disaster and part of the income tax paid in the past will be "refunded." In this regard, lifetime income taxation can play

[13]Camille Landais et al (2020) have called for a progressive European wealth tax to repay the COVID-19 debt. One may consider a one-time wealth tax to supplement the transition to lifetime taxation, although there will be an administrative challenge to implement it.

the role of insurance. Fourth, unlike annual income taxation, there is no incentive to delay the “realization” of income accrued during the current year to reduce taxation. Therefore, lifetime income taxation becomes neutral to the selection of income as capital gains, stock options, and retirement allowances.

Governments worldwide have increasingly relied on VAT to raise revenues. One of the reasons is that VAT is more compatible with economic growth and international competitiveness than other tax items. On the other hand, the redistribution the insurance (stabilization) functions of personal income tax system are becoming more important than ever because of the widening income inequality and destabilization of income, and to maintain these functions, it seems that there is a strong case to be made for converting annual income tax to lifetime income tax.

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Appendix 1: Tax Table (Function)

In the numerical example, I assume the following tax table. Note that in lifetime income taxation, the threshold income levels are defined in terms of the present value.

Taxable income	Marginal tax rate
～30,000	10%
30,001～60,000	20%
60,001～	35%

Appendix 2: Formal Model

In this appendix, I provide the formal model of lifetime income taxation to address its incentive effects. The lifetime income tax of this study is given as (A.1) where the cumulative average is discounted from the viewpoint of the initial period (j = 1 period), and is transformed to the current value (in period J) by the interest rate: .

$$\tau_J = (1+r)^{J-1}\left\{J * T\left(\frac{1}{J}\sum_{j=1}^{J}\frac{z_j}{(1+r)^{j-1}}\right) - (J-1) * T\left(\frac{1}{J-1}\sum_{j=1}^{J-1}\frac{z_j}{(1+r)^{j-1}}\right)\right\} \qquad \text{(A.1)}$$ [14]

τ_J denotes tax liability in period J, r is the real interest rate, which is assumed to be constant for expositional convenience, and z_j is real income that is realized in period j. By expressing taxable income in terms of present value, the tax base becomes neutral with respect to inflation. Suppose that the income of M_j is deferred from period j to the subsequent period and is saved at an interest rate of r

$$z_j = y_j + (1+r)M_{j-1} - M_j \qquad \text{(A.2)}$$

where y_j is accrued income in period j (the rate of return from deferred income may not be certain, but this will constitute the excess return in the lifetime income as is noted in footnote 13.) (A.2) may then be added with 1 + r being multiplied up to period J to write:

$$M_J = \sum_{j=1}^{J}(1+r)^{J-j}(y_j - z_j) \quad \text{or} \quad \sum_{j=1}^{J}\frac{z_j}{(1+r)^{j-1}} = \sum_{j=1}^{J}\frac{y_j}{(1+r)^{j-1}} - \frac{M_J}{(1+r)^{J-1}} \qquad \text{(A.2')}$$

The deferred income is realized sometime in the future, $M_N = 0$, where N stands for the last period. This implies that lifetime income on both accrual and realized basis are the same:

[14] As noted in the text, the tax in period J is less than the annual income tax:

$$\frac{T(z_J)}{J(1+r)^{J-1}} \geq \frac{1}{J}T\left(\frac{z_J}{(1+r)^{J-1}}\right) > T\left(\frac{1}{J}\frac{z_J}{(1+r)^{J-1}} + \left(1-\frac{1}{J}\right)\bar{z}_{J-1}\right) - \left(1-\frac{1}{J}\right)T(\bar{z}_{J-1})$$

⇔

$$T(z_J) > (1+r)^{J-1}\left\{J * T\left(\frac{1}{J}\frac{z_J}{(1+r)^{J-1}} + \left(1-\frac{1}{J}\right)\bar{z}_{J-1}\right) - (J-1)T(\bar{z}_{J-1})\right\}$$

given that the tax function is convex with $T'(z) > \mathrm{T}(z)/z$.

$$\sum_{j=1}^{N} \frac{y_j}{(1+r)^{j-1}} = \sum_{j=1}^{N} \frac{z_j}{(1+r)^{j-1}} \qquad \text{(A.3)}$$

By expressing (1) in terms of the discounted present value and adding lifetime tax liability over lifetime period, government tax collection is given by

$$\sum_{J=1}^{N} \frac{\tau_J}{(1+r)^{J-1}} = \frac{\tau_N}{(1+r)^N} = N * T\left(\frac{1}{N}\sum_{j=1}^{N} \frac{y_j}{(1+r)^{j-1}}\right) \qquad \text{(A.4)}$$

Let c_j be consumption in period j. The temporary household budget constraint (on an accrual basis) is given by

$$A_J - A_{J-1} = y_J + rA_{J-1} - \tau_J - c_J \qquad \text{(A.5)}$$

A_J refers to asset stock (including both safe and risky stock) in period J. Adding (A.5) and using (A.3), the lifetime budget constraint is

$$\sum_{j=1}^{N} \frac{c_j}{(1+r)^{j-1}} + \frac{A_N}{(1+r)^{N-1}} = \sum_{j=1}^{N} \frac{y_j}{(1+r)^{j-1}} - N * T\left(\frac{1}{N}\sum_{j=1}^{N} \frac{y_j}{(1+r)^{j-1}}\right) \qquad \text{(A.6)}$$

A_N is the bequest left that is included in the lifetime income of the recipients. (A.6) reveals that life income taxation serves as progressive consumption tax from a lifetime perspective. Relatedly, Band and Diamond (2008) briefly considered the equivalence of the non-linear setting provided that taxation is based on lifetime earnings and lifetime consumption.

In the following, I assume that the household earns a wage income and excess return from risky investment in their lifetime. The taxpayer may be confronted with wage shocks, random returns, and income shocks ε_J so that[15]

$$y_j = w_J L_J + (\rho_J - r)K_{J-1} + \varepsilon_J \qquad \text{(A.7)}$$

where w_J denotes the wage rate and K_J is the risky investment generating a rate of return ρ_J that is ex ante uncertain. The income shock may represent loss due to natural disasters and windfall gains from gifts and bequests. Labor supply is then decided after the wage rate is revealed, whereas K_J must be chosen before the return is known. Accordingly, $A_J - K_J$ is invested in safe assets.

[15] The household budget can be expressed on a realization basis by

$$S_J = z_J + (1+r)S_{J-1} - \tau_J - c_J$$

At this point, for generality assume that ΔM_{j-1} out of M_{j-1} the deferred income is invested in a risky asset. (A.2) becomes

$$z_j = y_j + (1+r)M_{j-1} + (\rho_J - r)\Delta M_{j-1} - M_j$$

Using (A.7), the realized income can be expressed as

$$z_j = w_J L_J + (\rho_J - r)(K_{J-1} + \Delta M_{j-1}) + (1+r)M_{j-1} - M_j + \varepsilon_J$$

Inserting the above into the budget yields

$$S_J + M_j = w_J L_J + (\rho_J - r)(K_{J-1} + \Delta M_{j-1}) + (1+r)(S_{j-1} + M_{j-1}) + \varepsilon_J - \tau_J - c_J$$

where S_J refers to the household's own savings inclusive of both risky and safe assets with $A_J = S_J + M_J$. Once K_{J-1} is optimized, ΔM_{j-1} becomes redundant and thus I can let $\Delta M_{j-1} = 0$. The household budget on a realization basis is reduced to (A.5).

The value function in period J is written as $V_J = V(A_{J-1}, M_{J-1}, K_{J-1}, G_{J-1})$. G_{J-1} is the discounted present value of accrual income up to period J, as defined below. Then the dynamic optimization of the household in the present context is expressed by

$$V(A_{J-1}, M_{J-1}, K_{J-1}, G_{J-1}) = Max_{\{c_J, L_J, A_J, K_J, M_J\}}\{U(c_J, L_J) + \beta E_J V(A_J, M_J, K_J, G_J)\}$$

subject to

$$c_J + A_J = w_J L_J + (\rho_J - r)K_{J-1} + \varepsilon_J + (1+r)A_{J-1} - \tau_J \quad : \lambda_J$$

and

$$\tau_J = (1+r)^{J-1}\left\{J * T\left(\frac{1}{J}G_J - \frac{M_J}{J(1+r)^{J-1}}\right) - (J-1) * T\left(\frac{1}{J-1}G_{J-1} - \frac{M_{J-1}}{(J-1)(1+r)^{J-2}}\right)\right\}$$

(A.1)

where

$$G_J = \sum_{j=1}^{J} \frac{1}{(1+r)^{j-1}}\left(w_j L_j + (\rho_j - r)K_{j-1} + \varepsilon_j\right) = \frac{1}{(1+r)^{J-1}}(w_J L_J + (\rho_J - r)K_{J-1}) + G_{J-1}$$

(A.8)

The constraint in the optimization combines (A.5) and (A.7), and λ_J denotes the Lagrange multiplier. The control variables are c_J, L_J, A_J, and M_j. The first-order conditions are given by

$$0 = U_{c_J} - \lambda_J \tag{A.9}$$

$$0 = U_{L_J} + \lambda_J w_J(1 - T_J') + \beta\left[E_J \frac{d}{dG_J} V_{J+1}\right]\frac{w_J}{(1+r)^{J-1}} \tag{A.10}$$

$$0 = -\lambda_J + \beta\left[E_J \frac{d}{dA_J} V_{J+1}\right] \tag{A.11}$$

$$0 = \beta\left[E_J \frac{d}{dK_J} V_{J+1}\right] \tag{A.12}$$

$$0 = \lambda_J T_J' + \beta\left[E_J \frac{d}{dM_J} V_{J+1}\right] \tag{A.13}$$

The derivatives of the value functions with respect to the state variables are expressed as:

$$\frac{d}{dA_{J-1}} V_J = \lambda_J(1+r) \tag{A.14}$$

$$\frac{d}{dM_{J-1}} V_J = -\lambda_J(1+r)T_{J-1}' \tag{A.15}$$

$$\frac{d}{dK_{J-1}} V_J = \lambda_J\left((\rho_J - r) - T_J'\right) + \beta\left[E_J \frac{\rho_J - r}{(1+r)^{J-1}} \frac{d}{dG_J} V_{J+1}\right] \tag{A.16}$$

$$\frac{1}{(1+r)^{J-1}} \frac{d}{dG_{J-1}} V_J = -\lambda_J(T_J' - T_{J-1}') + \beta(1+r)\left[E_J \frac{1}{(1+r)^J} \frac{d}{dG_J} V_{J+1}\right] \tag{A.17}$$

Combining (A.11) and (A.14) by moving forward one period yields

$$\lambda_J = \beta(1+r)E_J[\lambda_{J+1}] \qquad \text{(A.11')},$$

which in turn establishes the standard Euler equation as

$$U_{c_J} = \beta(1+r)\left[E_J U_{c_{J+1}}\right] \qquad \text{(A.9')}$$

(Tax Deferral)

Turn to the choice of the tax deferral. Replacing J-1 by J and substituting (A.15) into (A.13) yields

$$0 = \lambda_J T_J' - \beta(1+r)T_J'[E_J\lambda_{J+1}] \qquad \text{(A.13')}$$

The above is identical to (A.11') implying that M_j is redundant. Therefore, we can let $M_j = 0$, so $z_j = y_j$ for all period j in the following.

(Labor supply)

Now consider (A.17). It can be expressed as:

$$\frac{1}{(1+r)^J}\frac{d}{dG_J}V_{J+1} = -\lambda_{J+1}\left(T_{J+1}' - T_J'\right) + \beta(1+r)\left[E_{J+1}\frac{1}{(1+r)^{J+1}}\frac{d}{dG_{J+1}}V_{J+2}\right]$$

$$= -\lambda_{J+1}\left(T_{J+1}' - T_J'\right) + \beta(1+r)E_{J+1}\left[-\lambda_{J+2}\left(T_{J+2}' - T_{J+1}'\right) + E_{J+2}\frac{1}{(1+r)^{J+2}}\frac{d}{dG_{J+2}}V_{J+3}\right]$$

$$= -\sum_{j=J+1}^{N}(\beta(1+r))^{j-(J+1)}E_{J+1}\left[\lambda_j\left(T_j' - T_{j-1}'\right)\right]$$

$$= \lambda_{J+1}T_J' + \sum_{j=J+1}^{N-1}\left(\beta(1+r)\right)^{j-(J+1)}E_{J+1}\left[\beta(1+r)\lambda_{j+1} - \lambda_j\right]T_j'$$

$$- \left(\beta(1+r)\right)^{N-(J+1)}E_{t+1}\lambda_N T_N'$$

$$= \lambda_{J+1}T_J' - \left(\beta(1+r)\right)^{N-(J+1)}E_{J+1}\lambda_N T_N'$$

In the last equation, (A.11') is used with $E_{J+1}\left[E_j(*)\right] = E_{J+1}(*)$ for j>J+1. Inserting the above into (A.10) the following is established

$$0 = U_{L_J} + \lambda_J w_J\left(1 - T_J'\right) + \beta(1+r)E_J\left[\frac{1}{(1+r)^J}\frac{d}{dG_J}V_{J+1}\right]w_J$$

$$= U_{L_J} + w_J\left(U_{c_J} - \left(\beta(1+r)\right)^{N-J}E_J\lambda_N T_N'\right) \qquad \text{(A.10')}$$

Finally by solving (A.11') iteratively, we have:

$$\lambda_J = (\beta(1+r))^{N-J}E_J[\lambda_N] \qquad \text{(A.11'')}$$

Thus the first order condition for labor becomes:

$$0 = U_{L_J} + w_J U_{c_J}\left(1 - \frac{E_J\lambda_N T_N'}{E_J\lambda_N}\right) \qquad \text{(A.10'')}$$

(Risky investment)

Turn to (A.12), making use of (A.16),

$$0 = E_J\left[\lambda_{J+1}\left(\left(\rho_{J+1} - r\right) - T'_{J+1}\right)\right] + \beta(1+r)\left[E_J \frac{\rho_{J+1} - r}{(1+r)^{J+1}} \frac{d}{dG_{J+1}} V_{J+2}\right]$$

$$= E_J\left[\lambda_{J+1}\left(\left(\rho_{J+1} - r\right) - T'_{J+1}\right)\right]$$

$$+ \beta(1+r)E_J\left[\left(\rho_{J+1} - r\right)\left(\lambda_{J+2}T'_{J+1} - \left(\beta(1+r)\right)^{N-(J+2)}E_{J+1}\lambda_N T'_N\right)\right]$$

$$= E_J\left[\left(\rho_{J+1} - r\right)\lambda_{J+1}\left(1 - \frac{E_{J+1}\lambda_N T'_N}{E_{J+1}\lambda_N}\right)\right] \qquad \text{(A.12')}$$

In the last equality, (A.11'), replacing J with J+1 is used. To summarize:

Remarks:

The above proposition may be compared with the incentives under the conventional annual income taxation which solves:

$$V(A_{J-1}, M_{J-1}, K_{j-1}) = Max_{\{c_J, L_J, A_J, K_j\}}\left\{U(c_J, L_J) + \beta E_J V(A_J, M_J, K_j)\right\}$$

subject to

$$c_J + A_J = w_J L_J + (\rho_J - r)K_{J-1} + \varepsilon_J + (1+r)A_{J-1} - T(z_j) \quad : \lambda_J$$

where $z_j = y_j + (1+r)M_{j-1} - M_j$ .It can be established

$$0 = U_{L_J} + w_J U_{c_J}(1 - T'_J) \qquad \text{(A.18)}$$

$$0 = E_J\left[(\rho_{J+1} - r)\lambda_{J+1}(1 - T'_{j+1})\right] \qquad \text{(A.19)}$$

for labor and risky investment, which are standard. Regarding the tax deferral,

$$\lambda_J T'_j = \beta(1+r)\left[E_J \lambda_{j+1} T'_{j+1}\right] \qquad \text{(A.20)}$$

The above implies that the household aims to equalize the marginal tax rates over times. Indeed, without uncertainty, (A.20) reduces to:

$$T'_j = T'_{j+1} \qquad \text{for all j} \qquad \text{(A.20')}$$

Appendix 3: Insurance Function of the lifetime income taxation

The insurance function of lifetime income taxation can be seen from the simple two-period setting. Suppose a worker earns a constant income at y in the first period and continues to do so in a normal time in the second period. The behavioral response to the taxation is abstracted. In the second period, however, there may be a natural disaster or pandemic with the probability of q in which the worker suffers a complete loss of income. For simplicity we assume a zero-interest rate. Given that the income is constant in normal times, both conventional annual and lifetime income tax are presented by Eq. (1) levy $\mathrm{T}(y)$. In the case of a disaster, however, the annual income tax becomes $T(0) = 0$, whereas we have $\tau_2 = 2T(y/2) - T(y) < 0$. For analytical convenience, we linearly combine the two tax schemes so that $(1-\alpha)\mathrm{T}(0) + \alpha\tau_2 = \alpha\tau_2$ where $0 \leq \alpha \leq 1$. As α takes a higher value, the tax system is closer to the lifetime taxation. In the first period, the worker chooses an amount of savings as self-insurance. Note that with q=0, there is no need to save since the disposal income remains the same between the two periods. His optimization problem is expressed by

$$Max_{\{S\}}u(y - T(y) - S) + \{(1-q)u(y - T(y) + S) + qu(S - \alpha\tau_2)\}$$

that establishes

$$u(y - T(y) - S^*) = (1-q)u(y - T(y) + S^*) + qu(S^* - \alpha\tau_2) \qquad \text{(B.1)}$$

It is immediate to see that

$$\frac{d}{dq}S^* > 0 \text{ and } \frac{d}{d\alpha}S^* < 0 \qquad \text{(B.2)}$$

The saving increases in the probability of a disaster, but declines with α. In this context, the life time taxation works as insurance substituting the savings that are excessive in normal times. Now we can express the maximized utility as

$$\mathrm{W}(\mathrm{T}(y), \alpha, \tau_2, q) \equiv u(y - T(y) - S^*) + \{(1-q)u(y - T(y) + S^*) + qu(S^* - \alpha\tau_2)\} \quad \text{(B.3)}$$

Consider that tax reform directed toward lifetime taxation, raises α while keeping the expected tax revenue,

$$G = T(y) * (1 + 1 - q) + q\alpha\tau_2 \qquad \text{(B.4)}$$

constant by increasing $T(y)$ and leaving τ_2 unchanged (through adjusting $T(y/2)$). The welfare charged by

$$\mathrm{dW} \equiv -\{u_1' + (1-q)u_2'\}dT(y) + q(-\tau_2)\hat{u}_2' d\alpha \qquad \text{(B.5)}$$

where $u_1' = u'(y - T(y) - S^*)$, $u_2' = u(y - T(y) + S^*)$ and $\hat{u}_2' = u(S^* - \alpha\tau_2)$. Noting that

$$0 = \mathrm{dG} = (2 - \mathrm{q})\mathrm{d}T(y) + q\tau_2 d\alpha \qquad \text{(B.4')}$$

It can be established that the revenue neutral tax reform is welfare improving

$$\frac{d}{d\alpha}W|_{dG=0} \equiv q(-\tau_2)\left\{\hat{u}_2' - \frac{1}{2-q}\{u_1' + (1-q)u_2'\}\right\} > 0 \qquad \text{(B.5')}$$

with $u_1' > u_2' > \hat{u}_2'$ and $\tau_2 = 2T(y/2) - T(y) < 0$.

Appendix 4: Vickrey's Averaging Income Tax

In this appendix, I replicate the original proposal of averaging income tax by Vickrey (1939). It is different from this study in that interest income or normal return is counted, so income is defined in a comprehensive manner. Consider the following example. There are two taxpayers, Mr. A and Mr. B, who earn the same income $5,000 in each of the two periods. Vickrey (1939) supposes that they “start with the same capital, obtain the same rate of return” (page 382). In the present context, the capital can be regarded as an initial endowment. I distinguish between capital income from the endowment and interest income derived from the capital income. Mr. B can defer a part of his accrued income, say, \$2,500 in period 1 to period 2: the deterred income can be carried with an interest rate of 2%. Suppose that the same tax table as the main text applies. As a result, Mr. B pays less tax than Mr. A in period 1. Assume that both save all their disposable income at 2% interest rate. Indeed, Vickrey (1939) considers the two taxpayers “have identical earnings and expenditure during the period” (p. 382). In period 2, taxpayers collect their interest and deferred income. The incomes of Mr. A and Mr. B in period 2 are then \$50,860 and \$75,950, respectively. Vickrey (1939) adds tax payments in period 1 to interest to income. The reason is that the tax reduces savings (i.e., saving = accrued income — deferred income – tax), which in turn lowers interest income in period 2. Adding the tax in period 1 with interest works to recover interest income if the tax were not levied. Since consumption is zero in the first period, the total adjusted income is calculated as \$101,000 for both taxpayers. [16]

Realized income in period 1 + income in period 2 + interest× tax in period 1
= Income in period 1 - deferred income + Income in period 2 + deferred income
(1 + interest) +interest × (saving + tax in period 1)
= (1+ interest) × Income in period 1 + Income in period 2.

However, this approach may have a caveat. Consider Mr. C with the same lifetime income as the other two taxpayers. He does not defer income but consumes all in the first period. Consequently, income in period 2 becomes low without interest income, and so is adjusted income. Horizontal equity is then undermined. In contrast, this study calculates lifetime income directly to levy lifetime income taxation (that is equivalent to lifetime consumption taxation); thus, lifetime tax liability is independent of consumption pattern as well as realization of income.

[16] Arithmetically, the Vickery income averaging tax scheme can be expressed as

$$\tau_2 = T\big(z_2 + z_1 + r \times (S_1 + T(z_1))\big) - (1 + r)T(z_1)$$

with $\tau_1 = T(z_1)$, where S_1 is the saving in period 1 and assumed to be $S_1 = z_1 - T(z_1)$ in the table.

Period			Mr. A	Mr.B	Mr.C
1	Accruied income (1)		50,000	50,000	50,000
	Deferred income to period 2 (2)		0	25,000	0
	Realzied income (3)=(1)-(2)		50,000	25,000	50,000
	Tax (4)		7,000	2,500	7,000
	consumption (5)		0	0	43,000
	Saving =(3)-(4)-(5)		43,000	22,500	0
2	Income in period 2	Income (6)	50,000	50,000	50,000
		Deferred income from 1	0	25,500	0
		Interest income	860	450	0
		Sum (7)	50,860	75,950	50,000
	Interest*Tax in period 1 (8)		140	50	140
	Adjusted income (3)+(7)+(8)		101,000	101,000	100,140
	Tax (9)		23,350	23,350	23,049
	Net tax (10)= (9)-(1+0.02)*(4)		16,210	20,800	15,909
Lifetime	Income (1+0.02)*(1)+(6)		101,000	101,000	101,000

(Note) The calculation follows the tax return of footnote 5 (page 385) in Vickrey (1939).
(Note2) The deferred income is realized in the second period with interest.
(Note3)The same tax structure as Table 1 is assumed.

Appendix 5: Neutrality of Holding Period

The present lifetime income taxation assures the neutrality of the holding period associated with realization based capital taxation as addressed by Auerbach (1991) and Auerbach and Bradford(2004). Suppose the investor purchases an asset at a value of A_{J-1} in period J-1, which generates a return of ρ_J in period J, by holding it one more period, the investor can obtain ρ_{J+1}. Taxes applied when the asset is sold in periods J and J+1 can be denoted by τ_J and τ_{J+1}, respectively. Let t be tax on interest income or normal return. The arbitrage leads to

$$\left(1+\hat{\rho}_{J+1}\right)(1+\rho_J)A_{J-1}-\tau_{J+1}=(1+r(1-t))((1+\rho_J)A_{J-1}-\tau_J) \quad \text{(C.1)}$$

In the case of $\rho_{J+1}>\hat{\rho}_{J+1}$ the taxpayer chooses to hold the asset one more period whereas he would prefer to sell it if the inequality is reversed. The neutrality of the holding period requires $\hat{\rho}_{J+1}=r$. When $\hat{\rho}_{J+1}<r$, there arises a lock-in effect, where the investor is willing to keep an asset with a lower return so as to avoid taxes. Given that $\hat{\rho}_{J+1}=r$, (C.1) becomes:

$$\tau_{J+1}=\left(1+r(1-t)\right)\hat{\tau}_J+tr(1+\rho_J)A_{J-1} \quad \text{(C.2)}$$

Thus (C.2) is the condition of the tax structure required to eliminate the lock-in effect. Under conventional income taxation with $\tau_J=t\rho_J A_{J-1}$, the same tax rate being levied on capital gain and interest, (C.2) is re-written as

$$\tau_{J+1}=\left(1+r(1-t)\right)t\rho_J A_{J-1}+t\left\{(1+r)\left(1+\rho_J\right)-(1+\rho_J)\right\}A_{J-1}$$
$$=r(1-t)\tau_J+t\left\{\left(1+\hat{\rho}_{J+1}\right)\left(1+\rho_J\right)-1\right\}A_{J-1} \quad \text{(C.3)},$$

revealing that after-tax interest that will be charged on unpaid capital gain tax in period J alongside the taxation realized on capital gain in period J+1. A valuation problem will arise, however, it will be difficult for government to assess the intermediate value of the asset, i.e., $A_J = (1+\rho_J)A_{J-1}$ or equivalently ρ_J unless actual trading takes place.

To cope with the valuation problem regarding the initial and final cash flows, Auerbach (1991) proposes a retrospective capital tax, which is levied on the realized asset value so that $\tau_{J+1} = f_{J+1}A_{J+1}$ and $\tau_J = f_J A_J$. With $A_J = (1+\rho_J)A_{J-1}$, (C.2) can be re-written as

$$f_{J+1} = \big(1 + r(1-t)\big)f_J + tr \qquad \text{(C.4)}$$

Auerbach and Bradford (2004) generalizes the above and propose a cash flow capital tax: Like cash-flow taxation, tax on the sale of an asset and deduction on the purchase of the asset are made on each cash flow basis without reference to the asset's values or cash flows at other dates. Their cash flow tax is restricted to the linear whereas different tax rates are charged on sales and purchases of asset at different dates. In both (C.3) and (C.4), tax is charged on normal return r. In the case of $t = 0$, the condition of the neutrality turns to be trivial as (C.2) can reduce to

$$\tau_{J+1} = (1+r)\tau_J \qquad \text{(C.2')}$$

and can be fulfilled by a flat tax on capital gain or realization asset value, given that $\hat{\rho}_{J+1} = r$. Consider the lifetime income taxation. For comparison with Auerbach (1991), in this appendix, I take an aggregate lifetime income as tax base instead of averaging one. The tax payment thus becomes:

$$\tau_J = (1+r)^{J-1}\left\{T\left(\sum_{j=1}^{J}\frac{z_j}{(1+r)^{j-1}}\right) - T\left(\sum_{j=1}^{J-1}\frac{z_j}{(1+r)^{j-1}}\right)\right\} \qquad \text{(C.5)}$$

Note that the decision to sell the asset in period J and holding it for one more period generates the same lifetime value of the excess return when $\hat{\rho}_{J+1} = r$ as :

$$\frac{(1+r)(1+\rho_J)A_{J-1} - (1+r)^2 A_{J-1}}{(1+r)^J} = \frac{(1+\rho_J)A_{J-1} - (1+r)A_{J-1}}{(1+r)^{J-1}} \qquad \text{(C.6)}$$

Therefore, by abstracting other income for simplicity, the accumulated tax payment remains the same irrespective of the timing of realizing capital gain under the lifetime taxation, which assures the neutrality of holding period:

$$\frac{\tau_{J+1}}{(1+r)^J} = T\left(\frac{(1+\rho_J)A_{J-1} - (1+r)A_{J-1}}{(1+r)^{J-1}}\right) - T(0) = \frac{\tau_J}{(1+r)^{J-1}} \qquad \text{(C.7),}$$

with $\tau_j = 0$ in the other periods than the period when the asset is sold since the lifetime income does not change. The equation above also implies that (C.2') is satisfied.